



**Leonhard Euler**



**Daniel Bernoulli**

trave di EULER-BERNOULLI

IN 2D



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**Jacques Bernoulli**



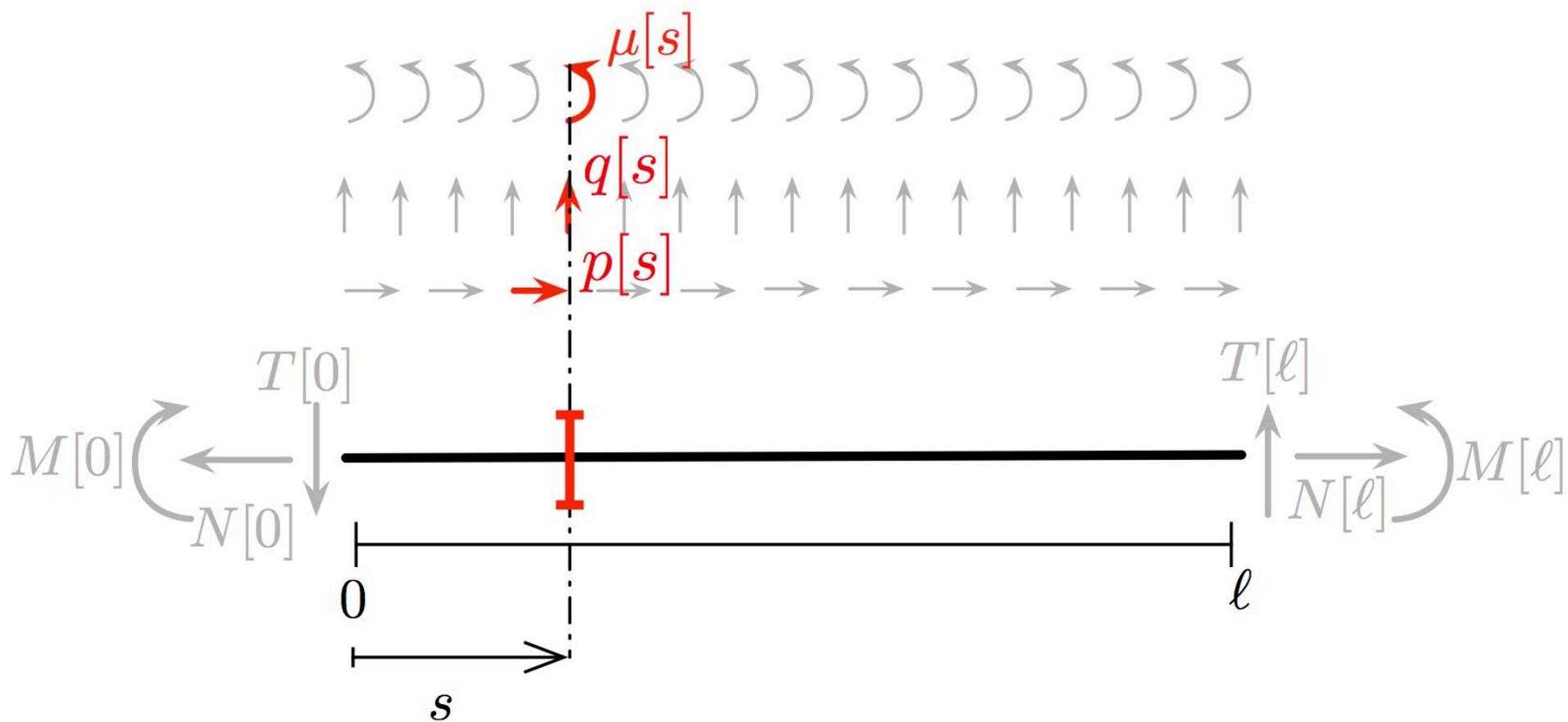
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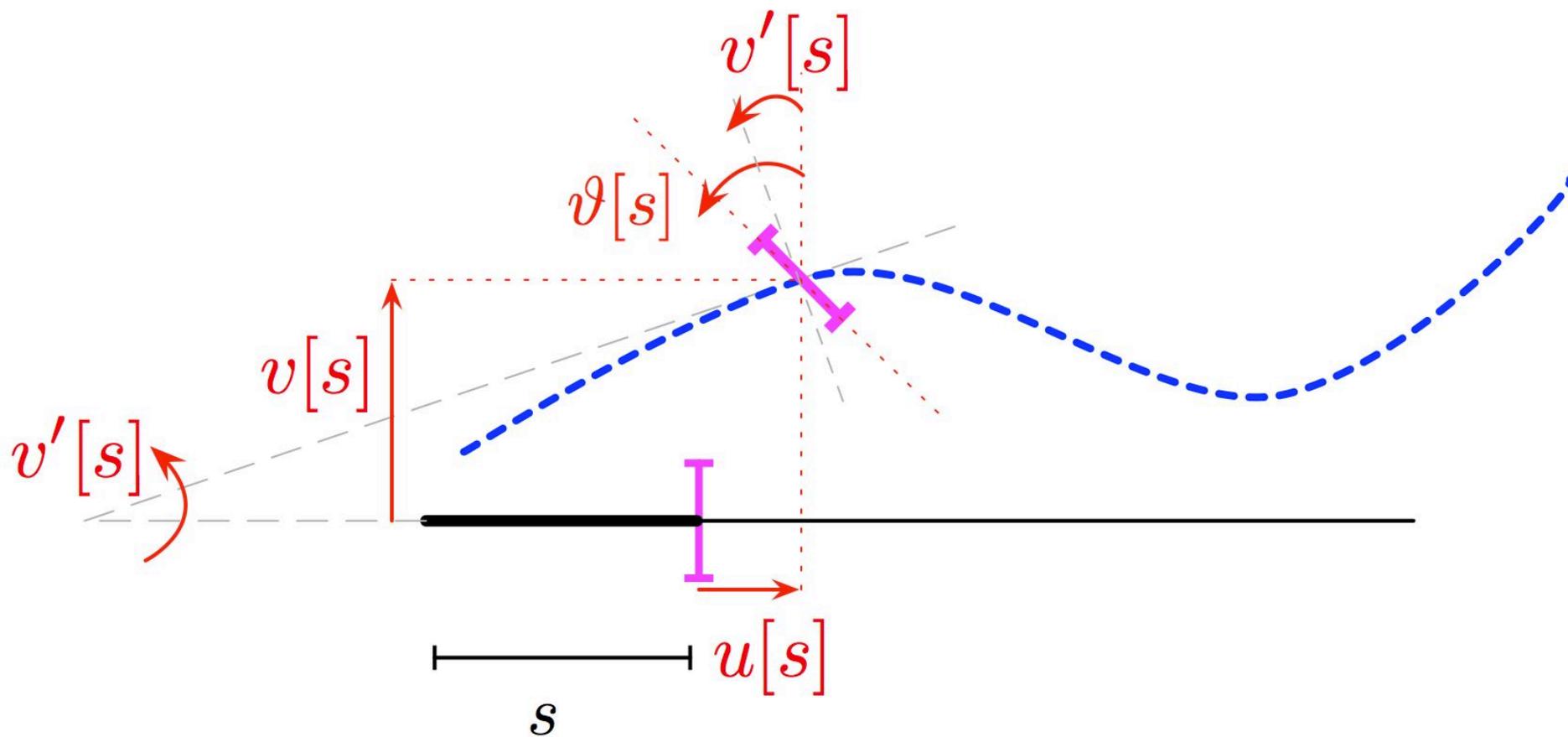
# dati del problema

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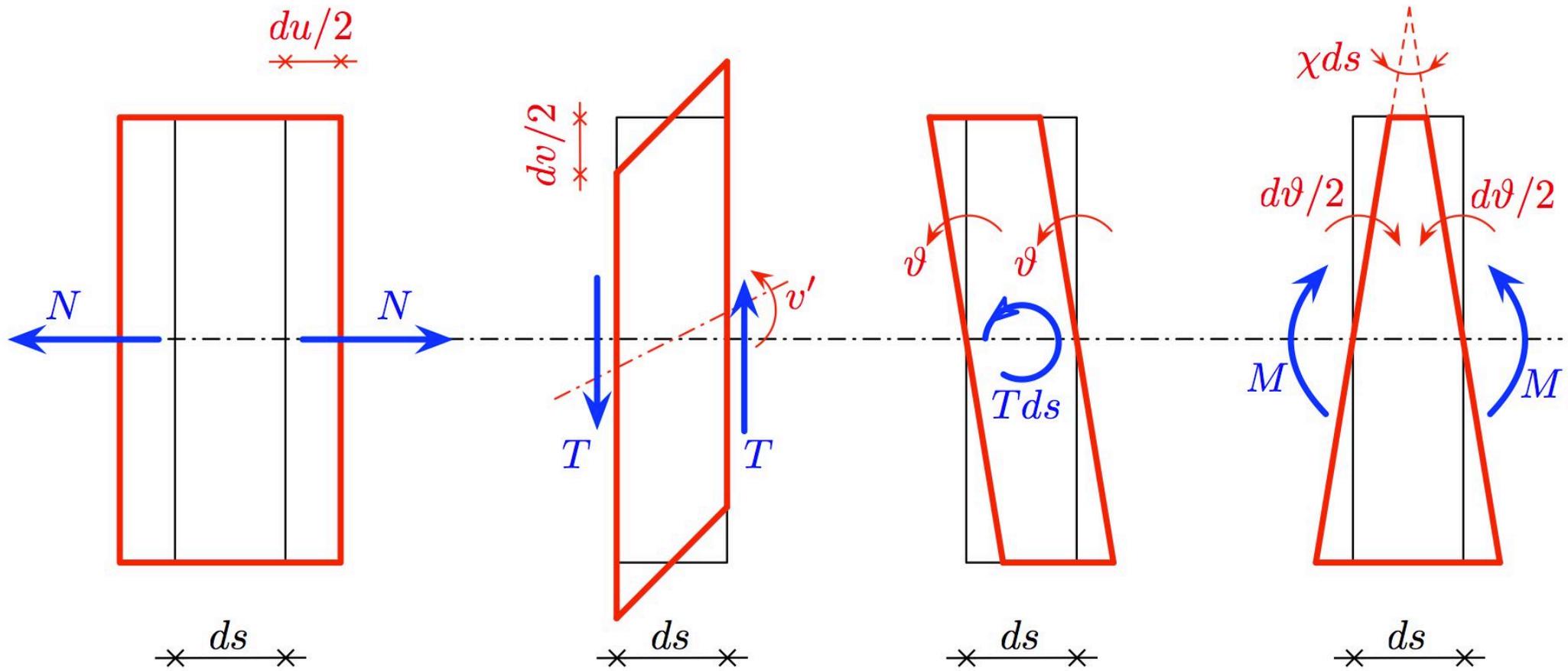


## cinematica: ipotesi di sezioni rigide

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# cinematica/dinamica interna: deformazioni/sollecitazioni



## equazioni del problema

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equilibrio

$$\begin{cases} N'[s] + p[s] = 0 \\ T'[s] + q[s] = 0 \\ M'[s] + \mu[s] + T[s] = 0 \end{cases}$$

compatibilità

$$\begin{cases} \varepsilon[s] = u'[s] \\ \gamma[s] = v'[s] - \vartheta[s] = \mathbf{0} \\ \chi[s] = \vartheta'[s] \end{cases}$$

legami costitutivi

$$\begin{cases} N[s] = EA\varepsilon[s] \\ M[s] = EJ\chi[s] \end{cases}$$

## equazioni del problema

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## equazioni del problema

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equilibrio

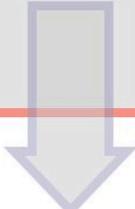
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equilibrio

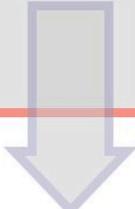
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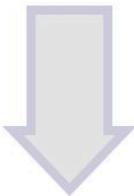
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$$\vartheta[s] = v'[s]$$


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## equazioni del problema

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equilibrio

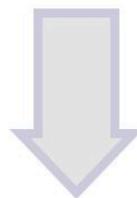
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$$\chi[s] = v''[s]$$

## equazioni del problema

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compatibilità

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$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

## equazioni del problema

---

equilibrio

$$N'[s] + p[s] = 0$$

$$M''[s] + \mu'[s] = q[s]$$

$$T[s] = -M'[s] - \mu[s]$$

compatibilità

$$\varepsilon[s] = u'[s]$$

$$\chi[s] = v''[s]$$

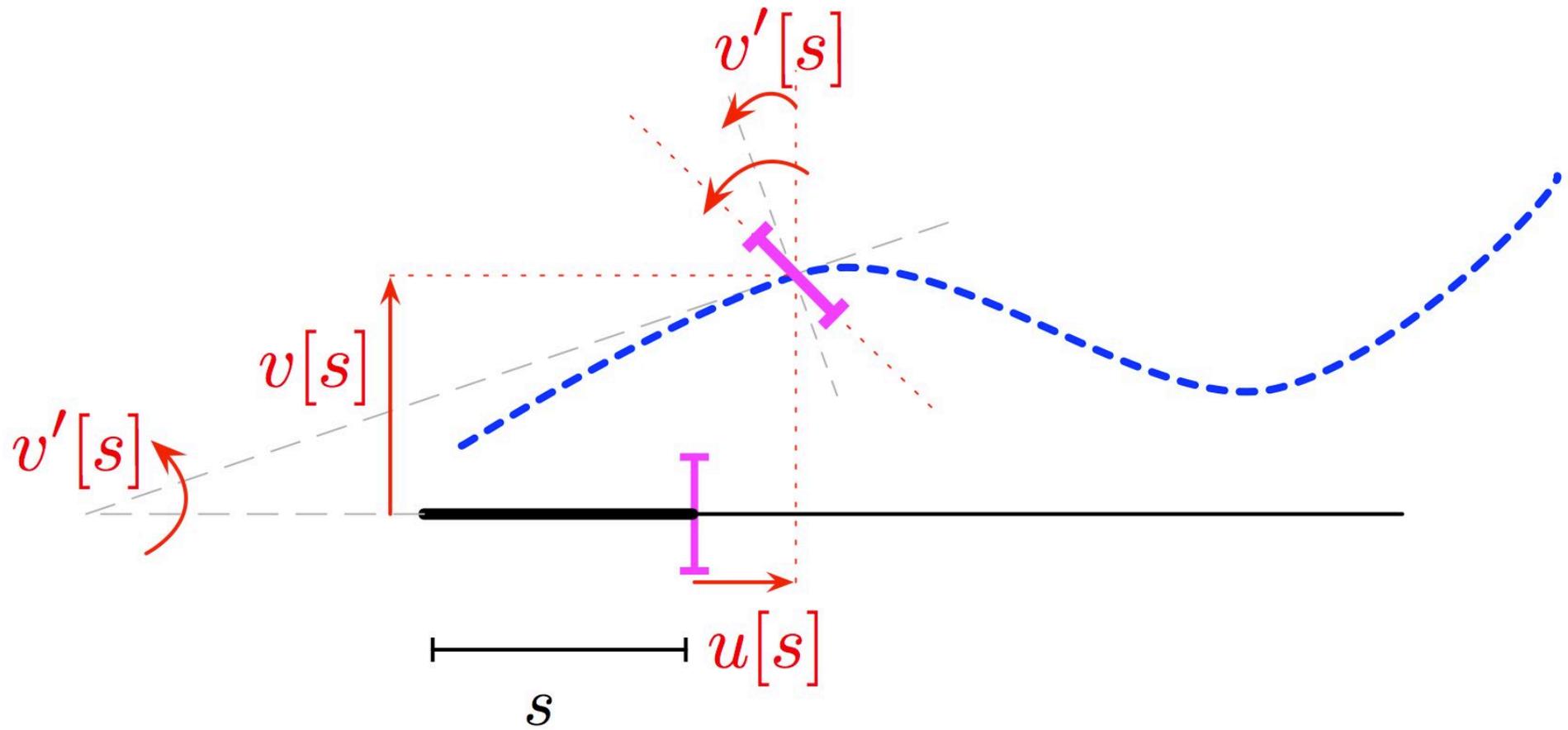
$$\vartheta[s] = v'[s]$$

legami costitutivi

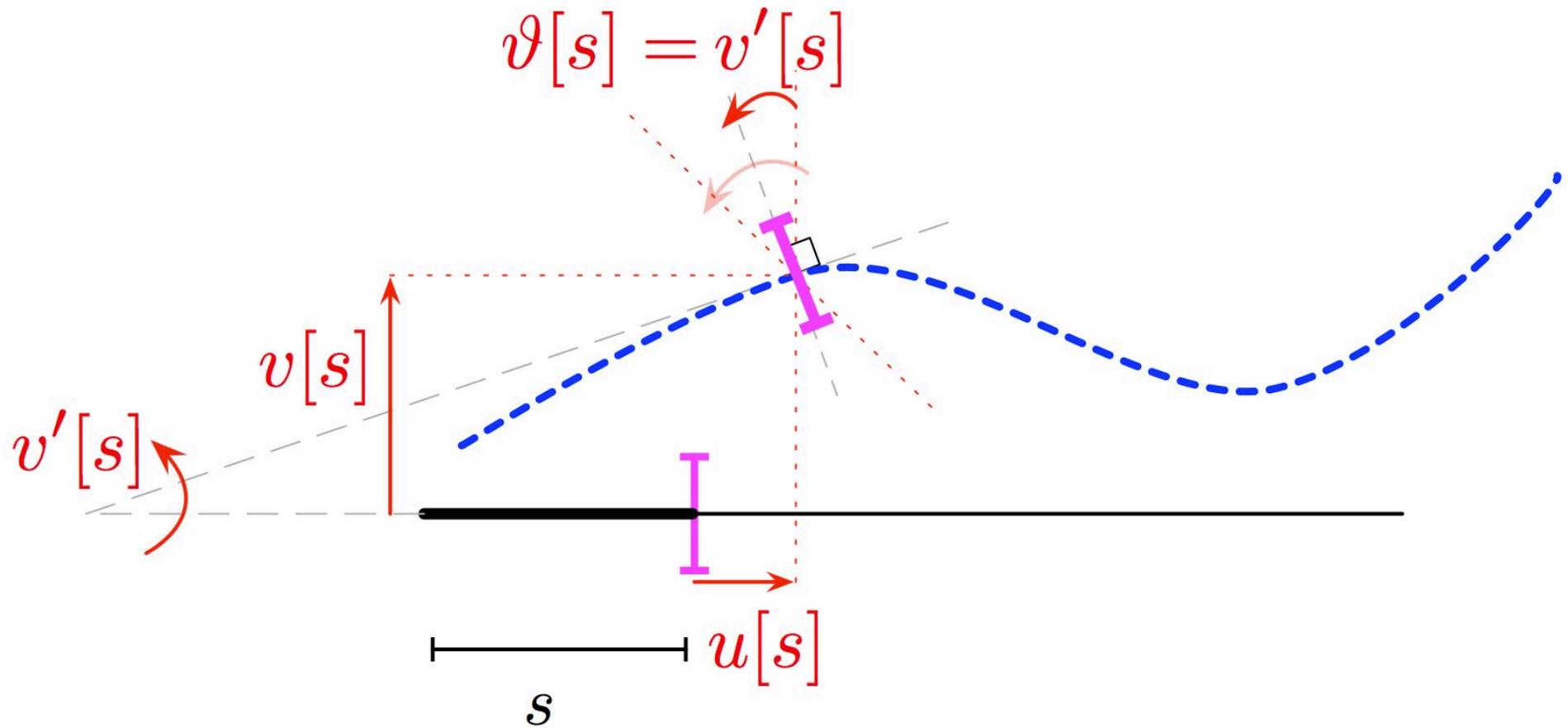
$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

# cinematica di Euler-Bernoulli

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cinematica di Euler-Bernoulli : **le sezioni si mantengono ortogonali all'(assetto dell')asse**



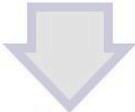
equazioni del problema **lineare-elastico della trave  
di Euler-Bernoulli**

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$$\begin{array}{l} \text{equilibrio} \\ T[s] = -M'[s] - \mu[s] \end{array} \left\{ \begin{array}{l} N'[s] + p[s] = 0 \\ M''[s] + \mu'[s] = q[s] \end{array} \right.$$
  
$$\begin{array}{l} \text{compatibilità} \\ \vartheta[s] = v'[s] \end{array} \left\{ \begin{array}{l} \varepsilon[s] = u'[s] \\ \chi[s] = v''[s] \end{array} \right.$$
  
$$\begin{array}{l} \text{legami} \\ \text{costitutivi} \end{array} \left\{ \begin{array}{l} N[s] = EA\varepsilon[s] \\ M[s] = EJ\chi[s] \end{array} \right.$$

equazioni del problema **lineare-elastico della trave  
di Euler-Bernoulli**

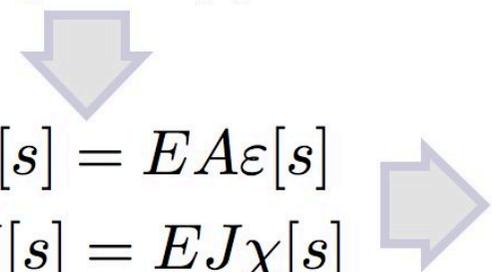
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equilibrio	{	$N'[s] + p[s] = 0$
$T[s] = -M'[s] - \mu[s]$		$M''[s] + \mu'[s] = q[s]$
compatibilità	{	$\varepsilon[s] = u'[s]$
$\vartheta[s] = v'[s]$		$\chi[s] = v''[s]$
		
legami	{	$N[s] = EA\varepsilon[s]$
costitutivi		$M[s] = EJ\chi[s]$

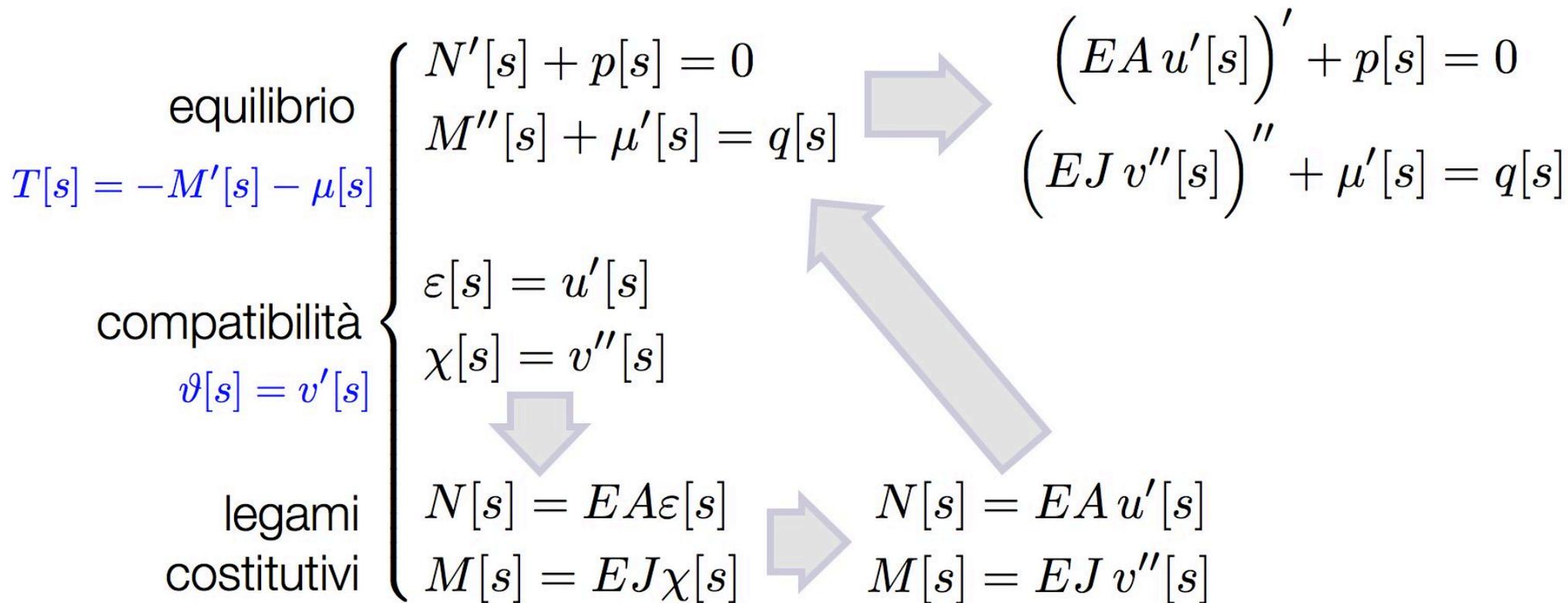
equazioni del problema **lineare-elastico della trave di Euler-Bernoulli**

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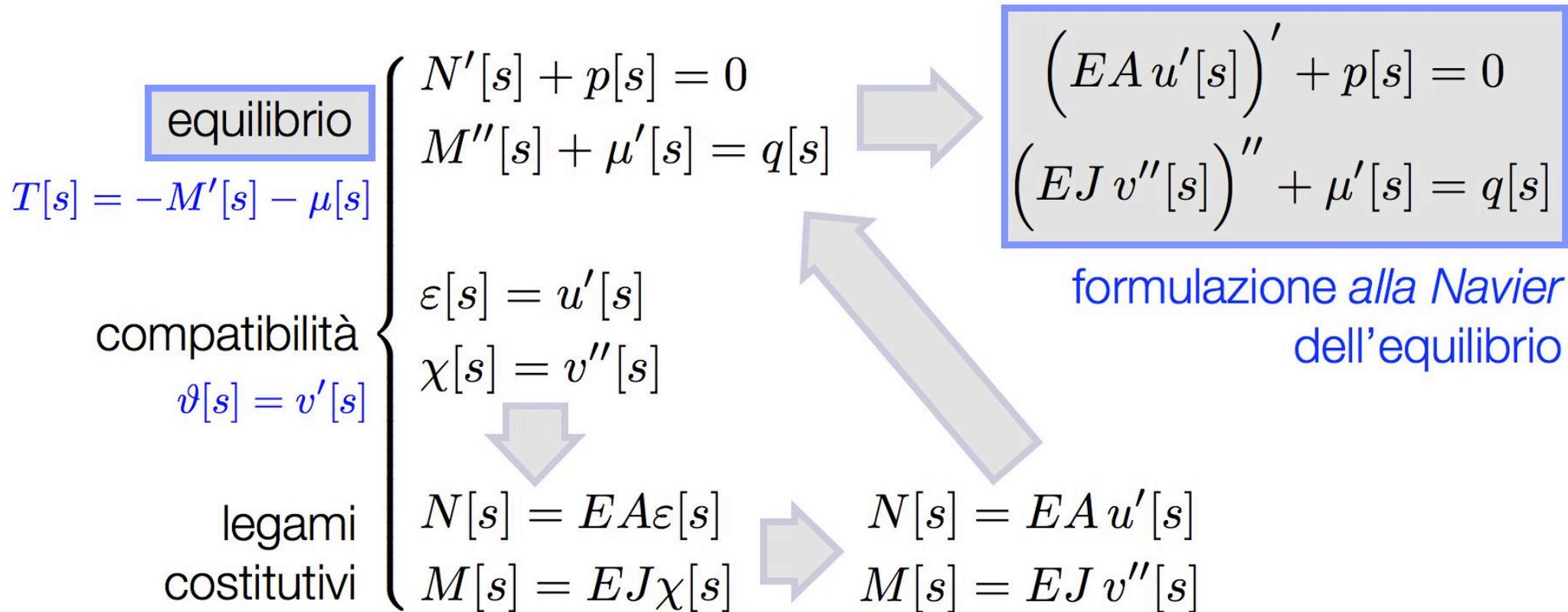
equilibrio	}	$N'[s] + p[s] = 0$	
$T[s] = -M'[s] - \mu[s]$		$M''[s] + \mu'[s] = q[s]$	
compatibilità	}	$\varepsilon[s] = u'[s]$	
$\vartheta[s] = v'[s]$		$\chi[s] = v''[s]$	
legami costitutivi	}	$N[s] = EA\varepsilon[s]$	$N[s] = EA u'[s]$
		$M[s] = EJ\chi[s]$	$M[s] = EJ v''[s]$



equazioni del problema **lineare-elastico della trave di Euler-Bernoulli**



equazioni del problema **lineare-elastico della trave di Euler-Bernoulli**



equazioni del problema **lineare-elastico della trave  
di Euler-Bernoulli**

---

$$\begin{aligned} \left( EA u'[s] \right)' + p[s] &= 0 \\ \left( EJ v''[s] \right)'' + \mu'[s] &= q[s] \end{aligned}$$

- il problema di equilibrio si presenta come un sistema di 2 equazioni **differenziali**
- le equazioni sono **disaccoppiate**, problema *assiale* e *flessionale*
- si tratta di **equazioni di campo**:  
le incognite sono *campi (funzioni dello spazio)*
- la soluzione è una funzione **polinomiale** in  $s$

equazioni del problema **lineare-elastico della trave**  
**di Euler-Bernoulli**

---

$$\left( EA u'[s] \right)' + p[s] = 0$$
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equazioni del problema **lineare-elastico della trave**  
**di Euler-Bernoulli**

---

$$\begin{aligned} (EA u'[s])' + p[s] &= 0 \\ (EJ v''[s])'' + \mu'[s] &= q[s] \end{aligned}$$

$\Rightarrow u[s], v[s] \quad \Rightarrow \begin{aligned} \varepsilon[s] &= u'[s] \\ \chi[s] &= v''[s] \end{aligned} \quad \Rightarrow \begin{aligned} N[s] &= EA u'[s] \\ M[s] &= EJ v''[s] \end{aligned}$

equazioni del problema **lineare-elastico della trave**  
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$$\begin{aligned} \left( EA u'[s] \right)' + p[s] &= 0 \\ \left( EJ v''[s] \right)'' + \mu'[s] &= q[s] \end{aligned}$$

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equazioni del problema **lineare-elastico della trave**  
**di Euler-Bernoulli**

trave a sezione  
costante

$$\begin{aligned} (EA u'[s])' + p[s] &= 0 \\ (EJ v''[s])'' + \mu'[s] &= q[s] \end{aligned}$$



$$\begin{aligned} EA u''[s] + p[s] &= 0 \\ EJ v''''[s] + \mu'[s] &= q[s] \end{aligned}$$



$$u[s], v[s]$$

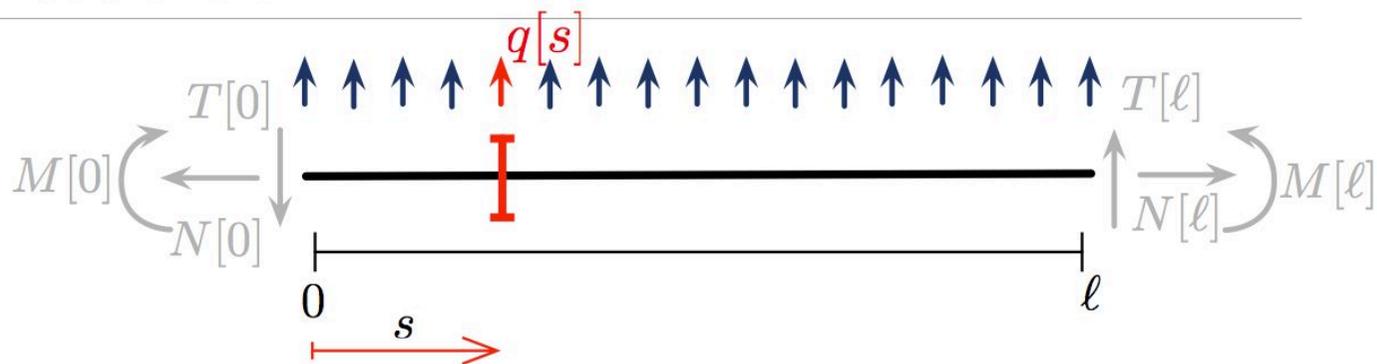


$$\begin{aligned} \varepsilon[s] &= u'[s] \\ \chi[s] &= v''[s] \\ \vartheta[s] &= v'[s] \end{aligned}$$



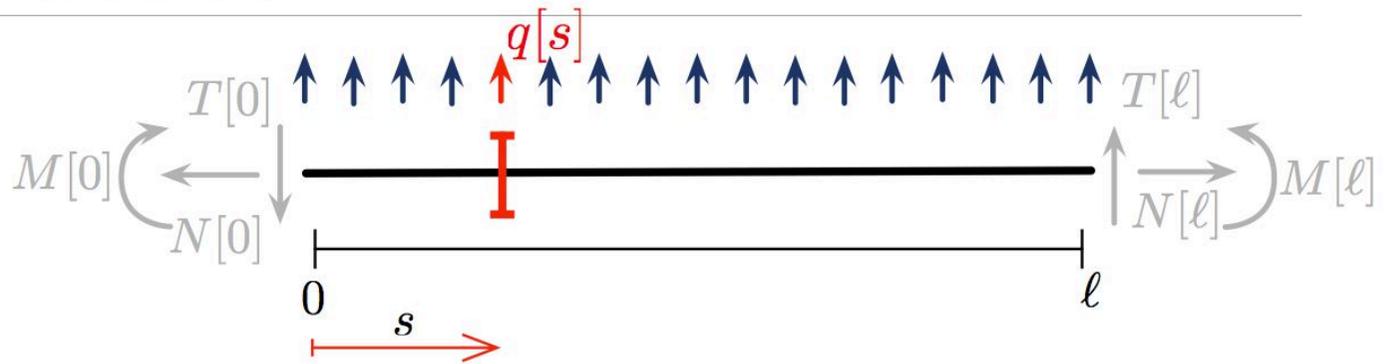
$$\begin{aligned} N[s] &= EA u'[s] \\ M[s] &= EJ v''[s] \\ T[s] &= -M'[s] - \mu[s] \\ &= -\left(EJ v''[s]\right)' - \mu[s] \end{aligned}$$

# esempio di problema flessionale



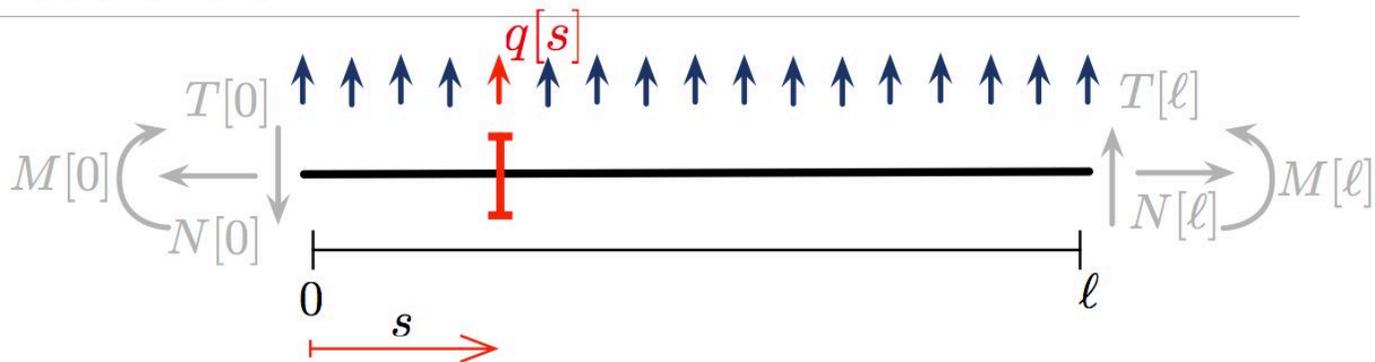
esempio di problema flessionale

$$EJ v''''[s] = q[s]$$



esempio di problema flessionale

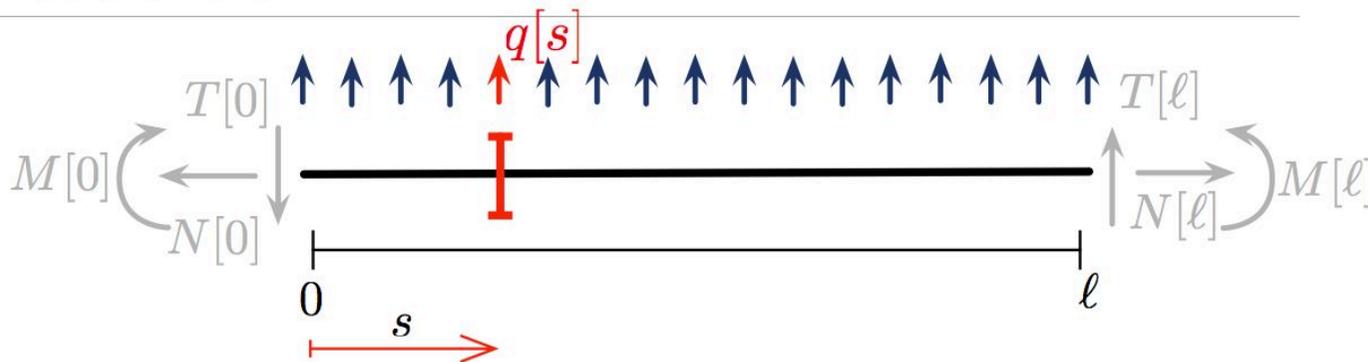
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$v[s]$  polinomio almeno di ordine 4 in  $s$

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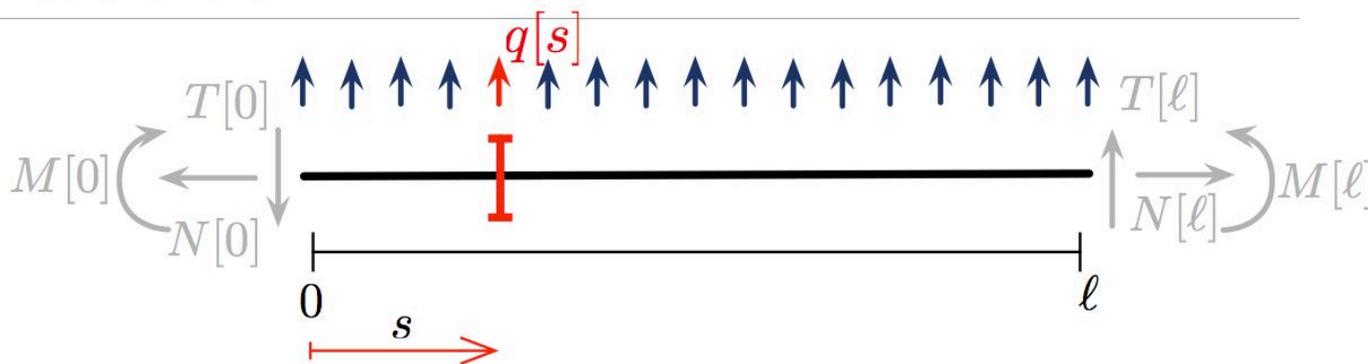


$v[s]$  polinomio almeno di ordine 4 in  $s$

$$v[s] = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + \dots$$

## esempio di problema flessionale

$$EJ v''''[s] = q[s]$$

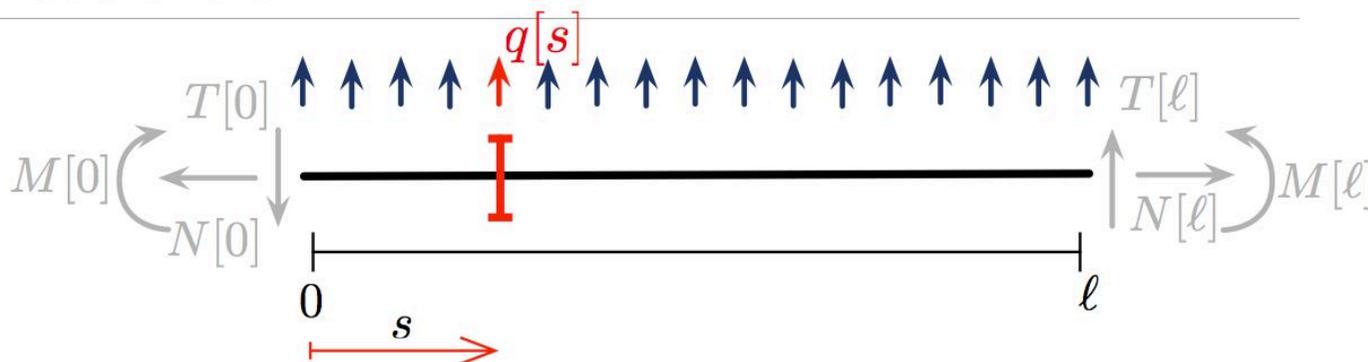


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$$\begin{aligned} v[s] &= c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + \dots \\ v'[s] &= c_1 + 2c_2 s + 3c_3 s^2 + 4c_4 s^3 + \dots \end{aligned}$$

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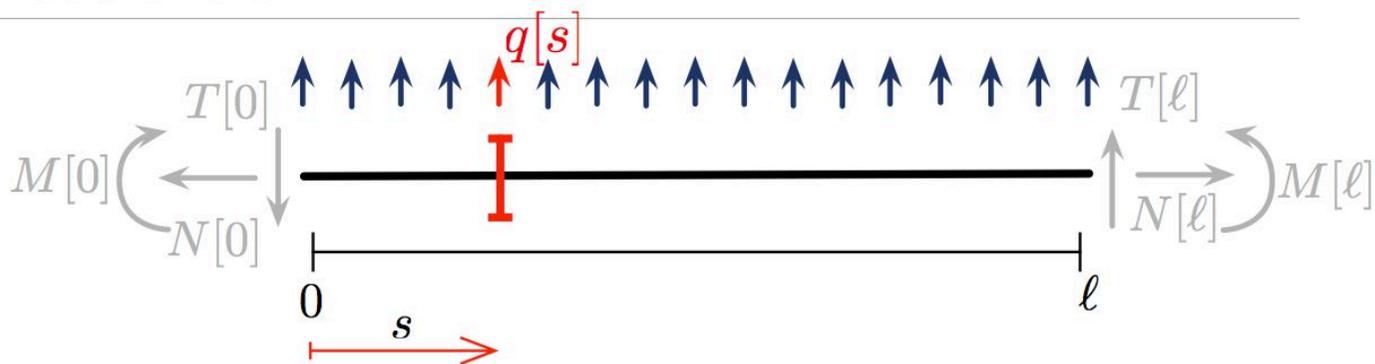


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## esempio di problema flessionale

$$EJ v''''[s] = q[s]$$

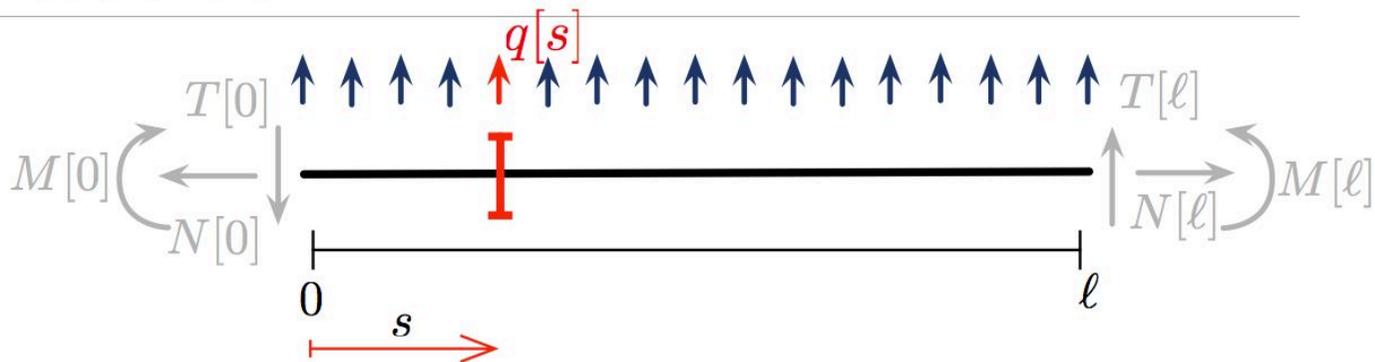


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## esempio di problema flessionale

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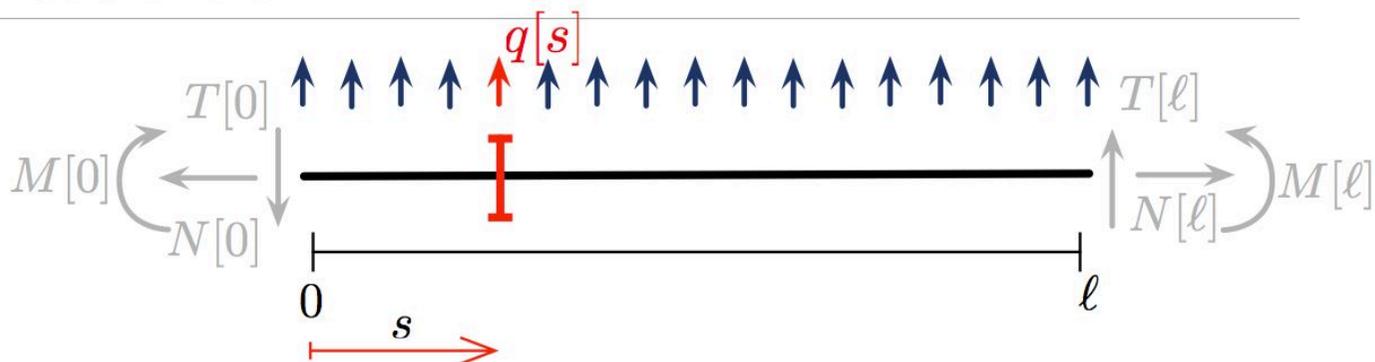


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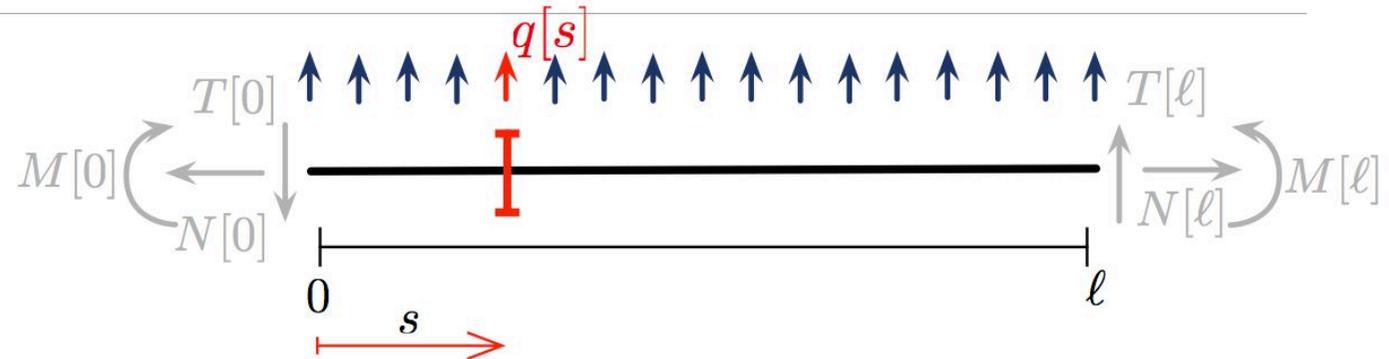


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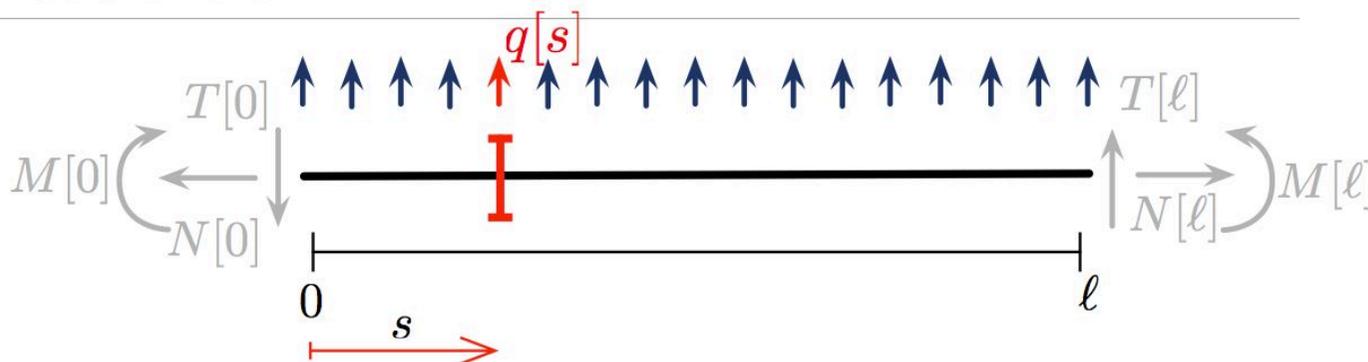
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termini noti in base a  $q[s]$

## esempio di problema flessionale

$$EJ v''''[s] = q[s]$$



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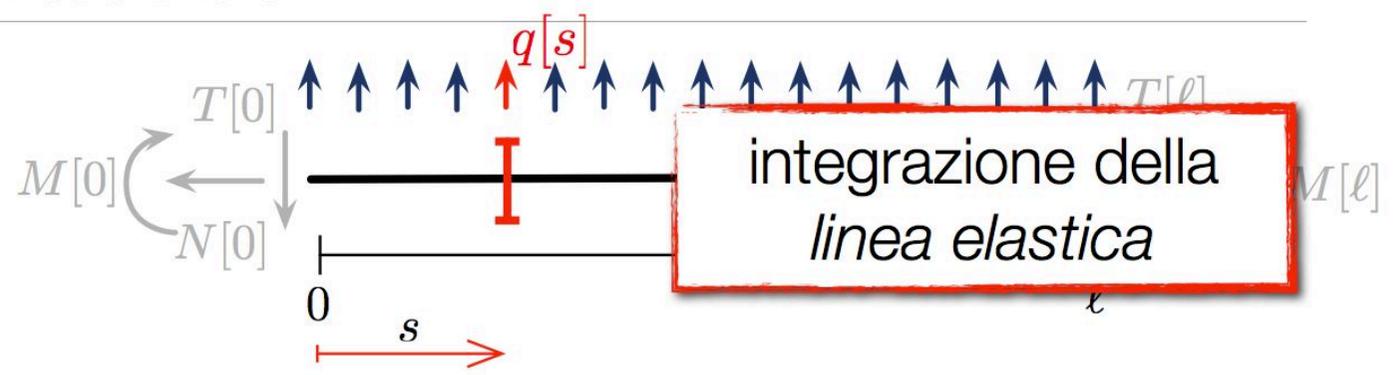
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 \end{aligned}$$

termini noti in base alle  
condizioni al bordo  $[0, \ell]$

termini noti in  
base a  $q[s]$

## esempio di problema flessionale

$$EJ v''''[s] = q[s]$$



$v[s]$  polinomio almeno di ordine 4 in  $s$

$v[s] =$	$c_0$	+	$c_1$	$s$	+	$c_2$	$s^2$	+	$c_3$	$s^3$	+	$c_4$	$s^4$	+	$\dots$
$v'[s] =$			$c_1$			$2c_2$	$s$		$3c_3$	$s^2$		$4c_4$	$s^3$		$\dots$
$v''[s] =$						$2c_2$			$6c_3$	$s$		$12c_4$	$s^2$		$\dots$
$v'''[s] =$									$6c_3$			$24c_4$	$s$		$\dots$
$v''''[s] =$												$24c_4$			$\dots$

termini noti in base alle  
condizioni al bordo  $[0, \ell]$

termini noti in  
base a  $q[s]$

PROGETTO e VERIFICA

a presso/tenso-flessione

## stato tensionale per presso/tenso-flessione

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legami costitutivi

$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

## stato tensionale per presso/tenso-flessione

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legami costitutivi

$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

deformazione

$\varepsilon, \chi$



sollecitazione

$N, M$

## stato tensionale per presso/tenso-flessione

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$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

deformazione

$\varepsilon, \chi$



sollecitazione

$N, M$

- modello di fibre estensibili

## stato tensionale per presso/tenso-flessione

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$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

- modello di fibre estensibili

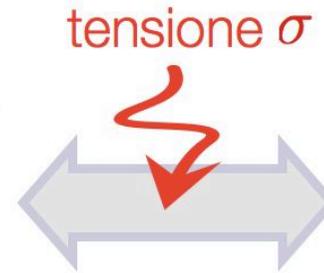


# stato tensionale per presso/tenso-flessione

legami costitutivi

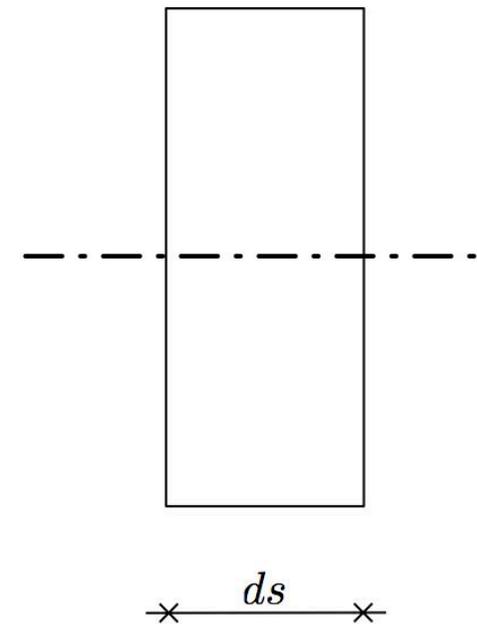
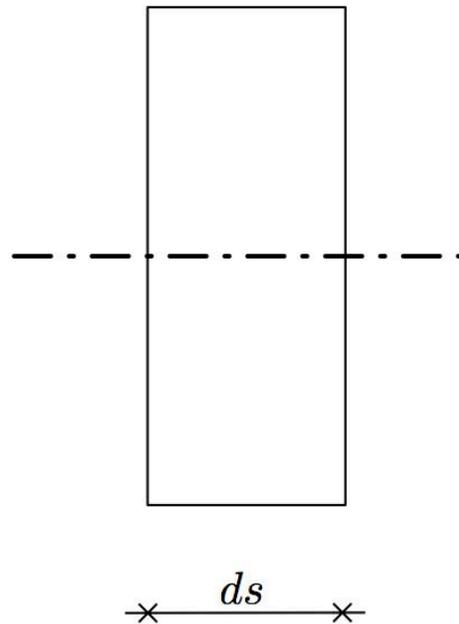
$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$

deformazione  
 $\varepsilon, \chi$



sollecitazione  
 $N, M$

- modello di fibre estensibili

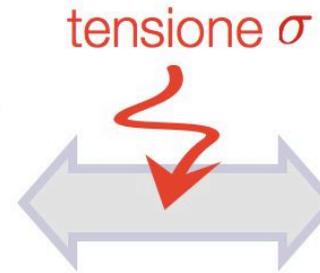


# stato tensionale per presso/tenso-flessione

legami costitutivi

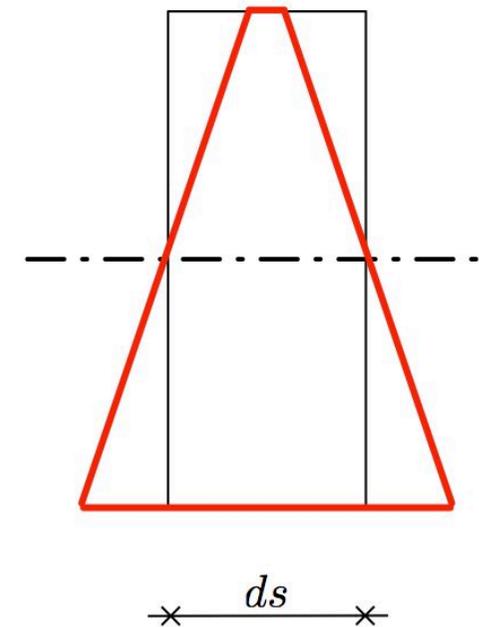
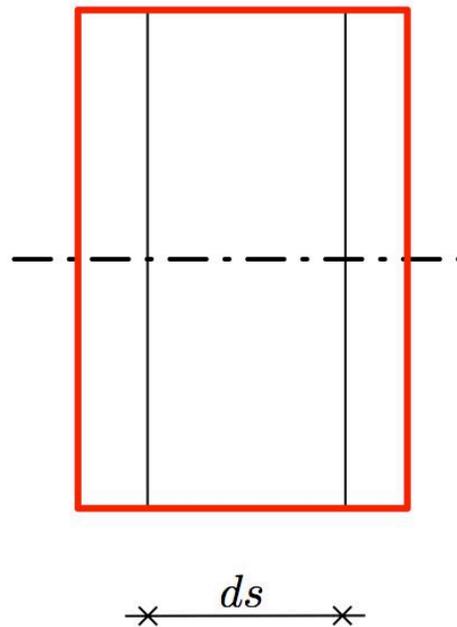
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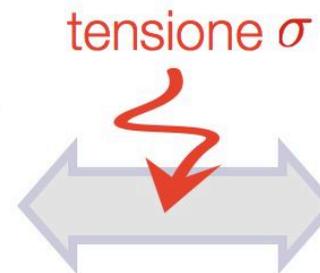


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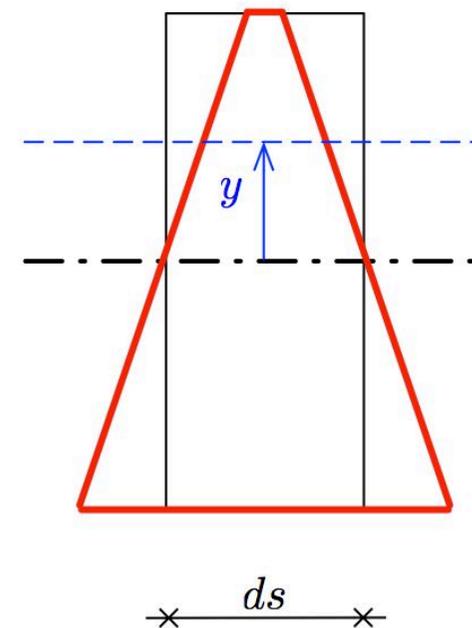
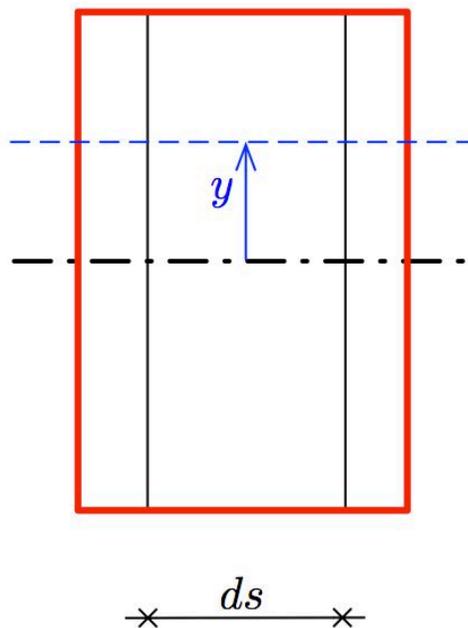
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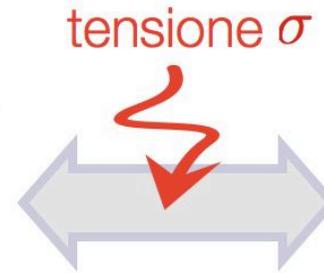


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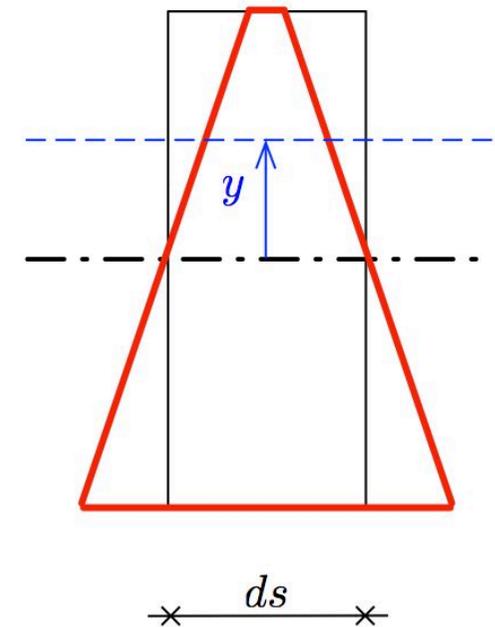
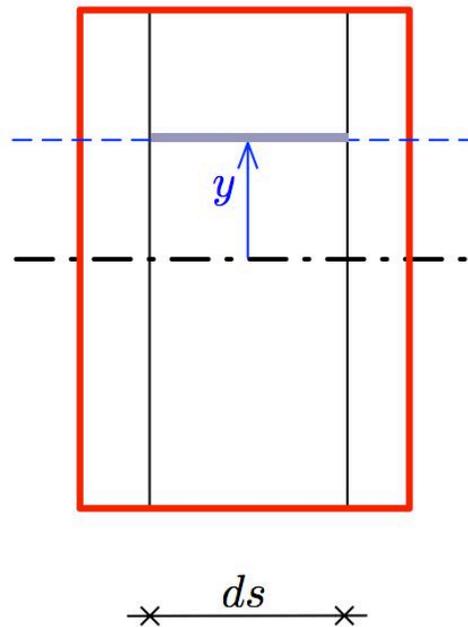
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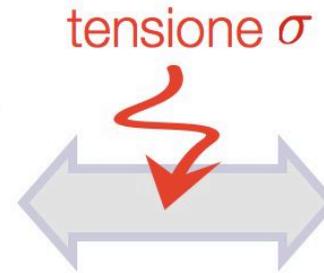


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legami costitutivi

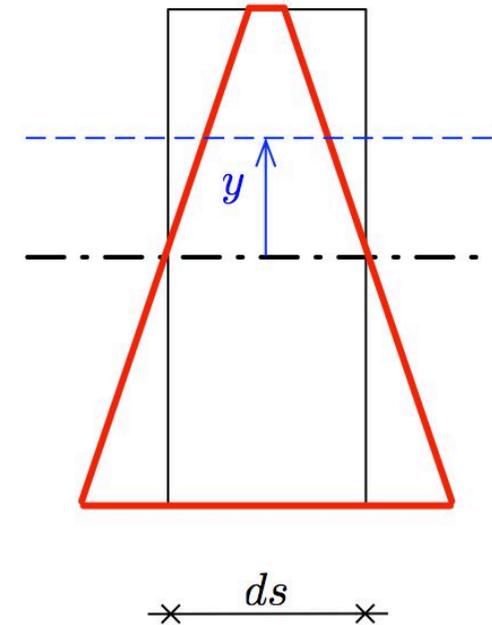
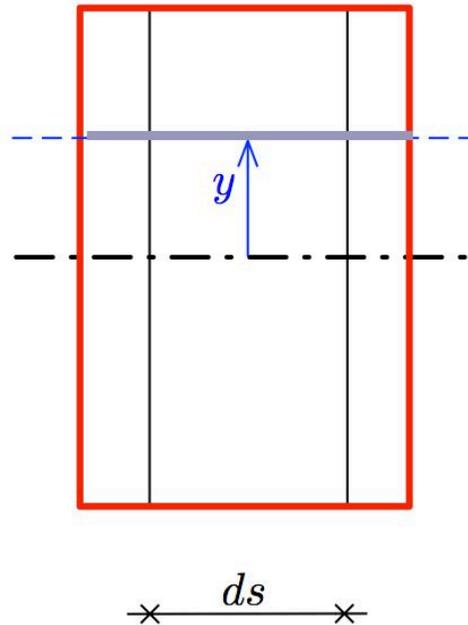
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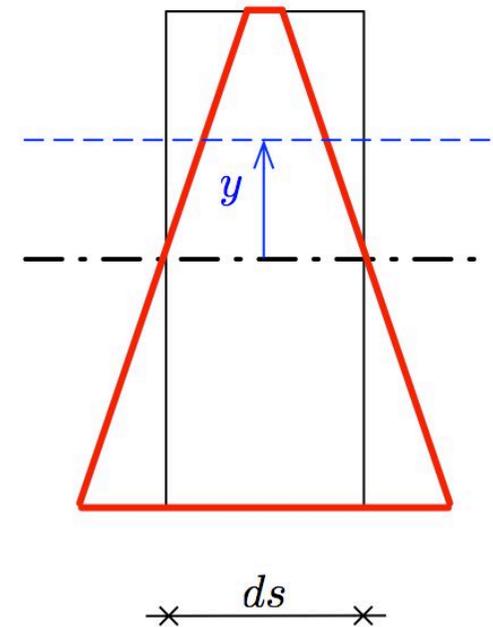
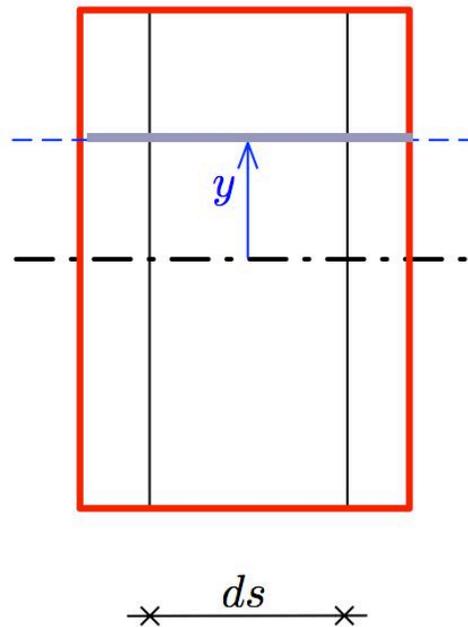
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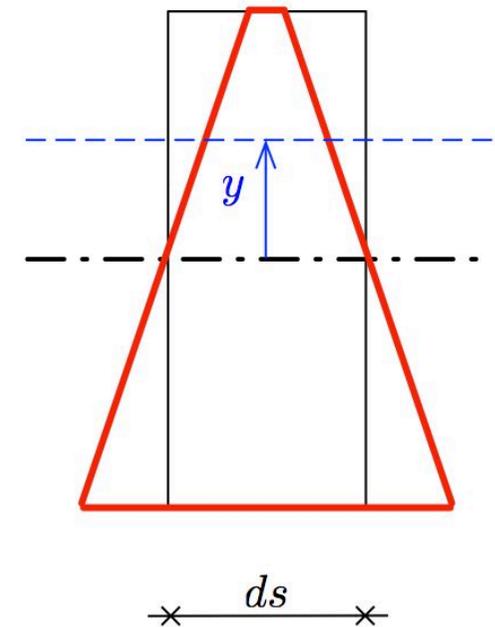
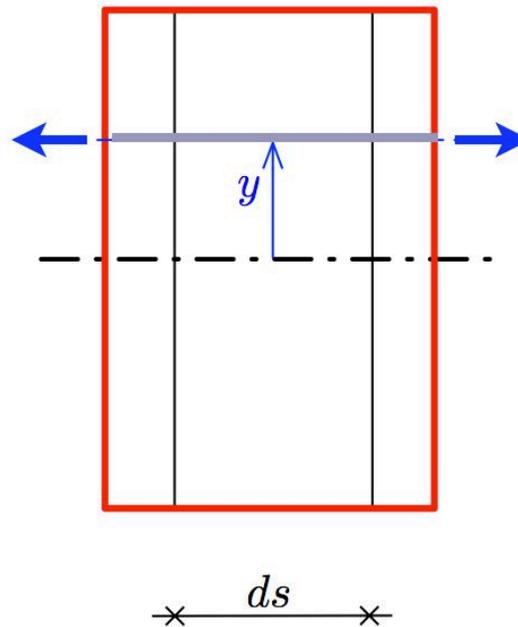
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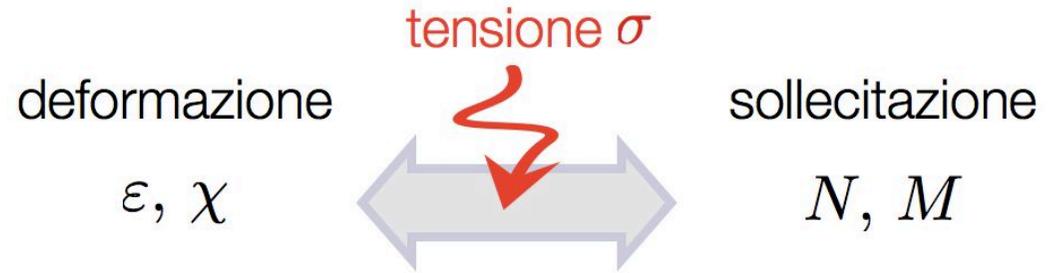


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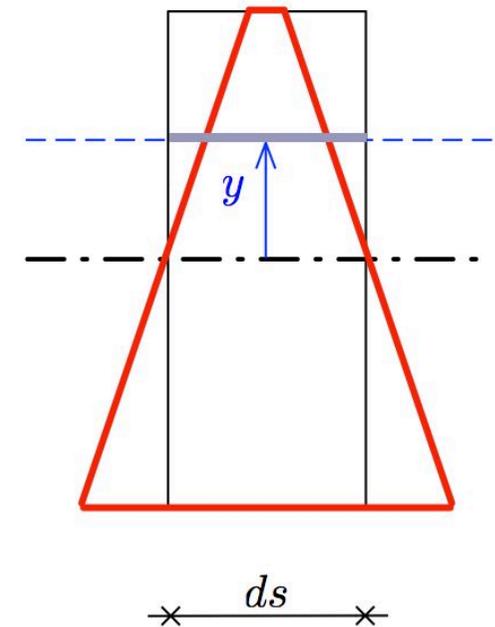
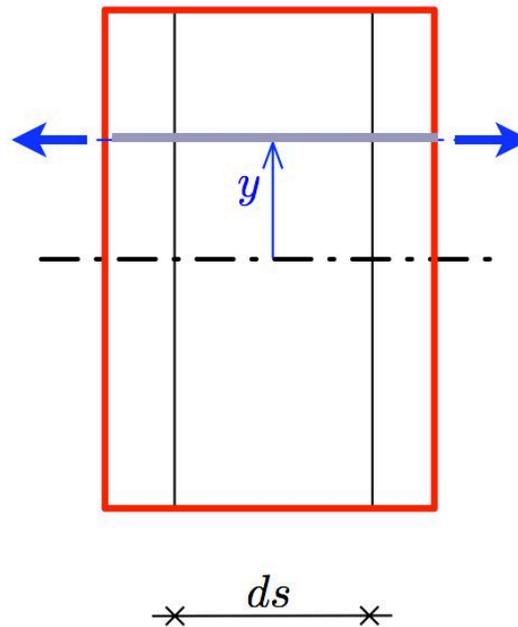
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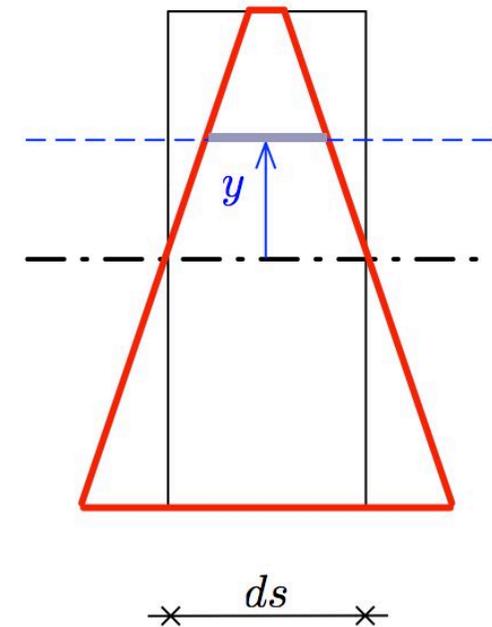
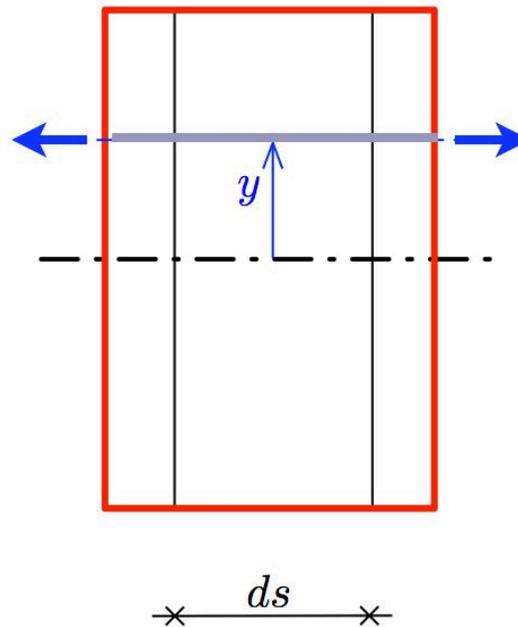
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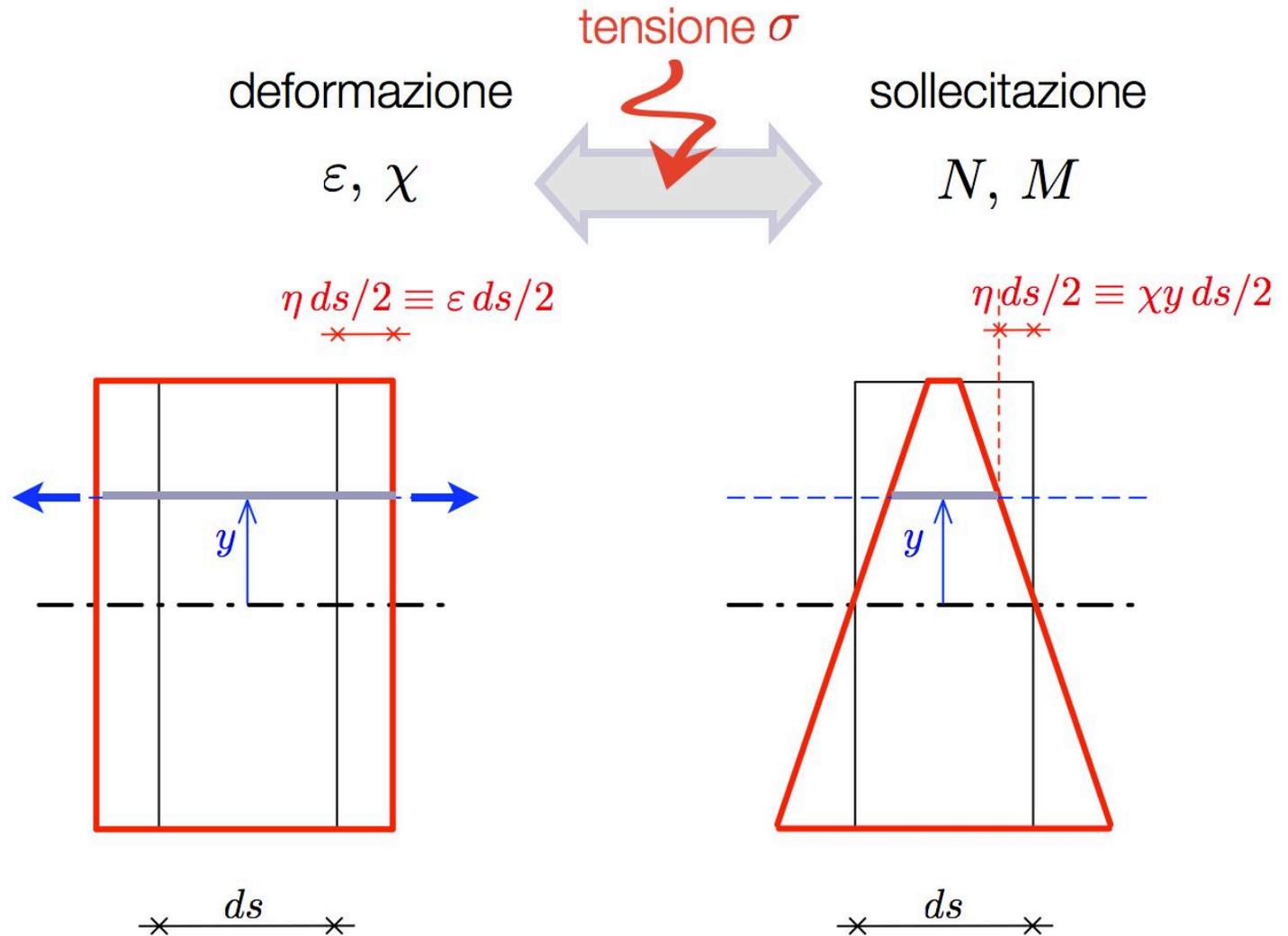


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legami costitutivi

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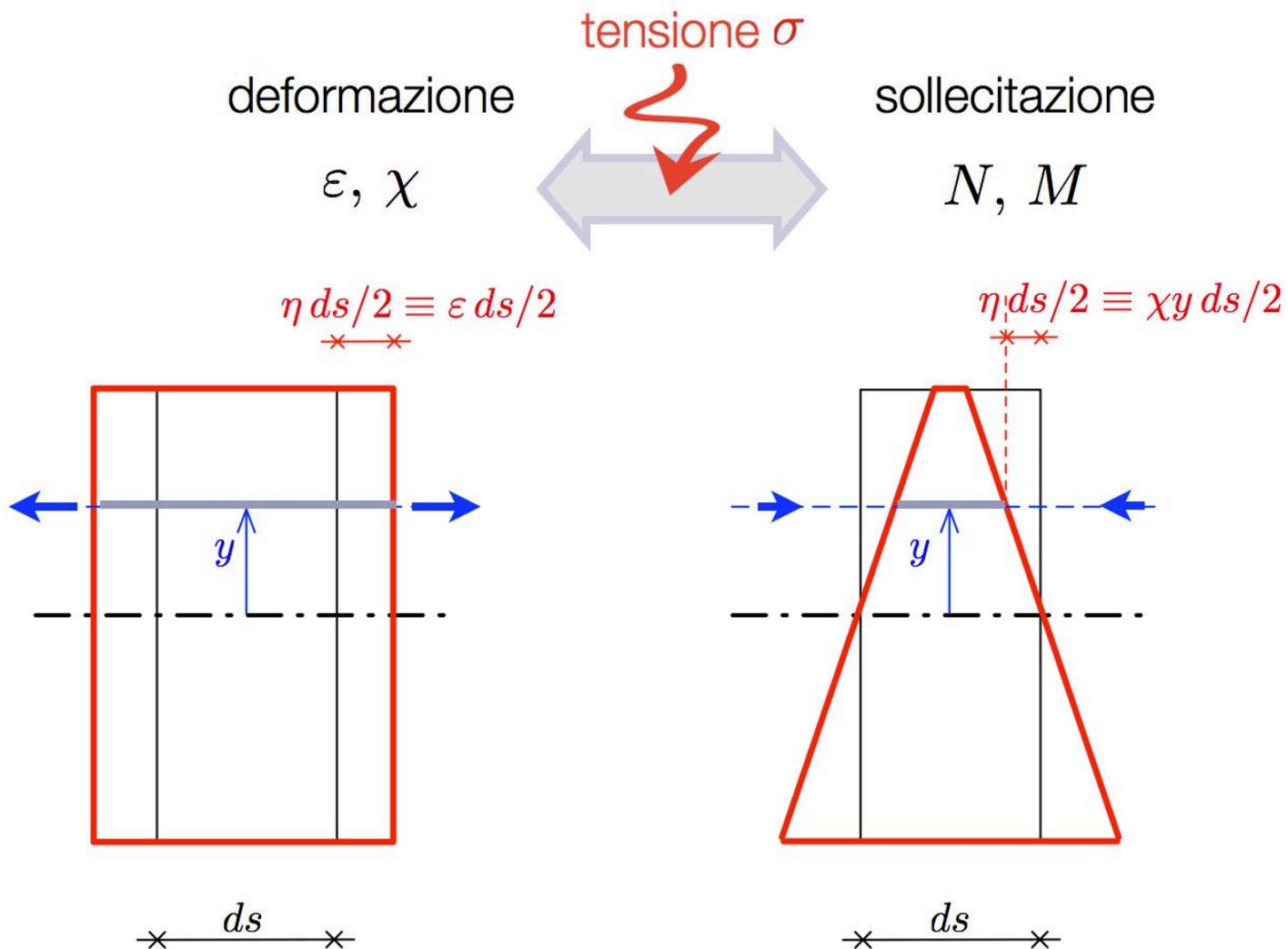


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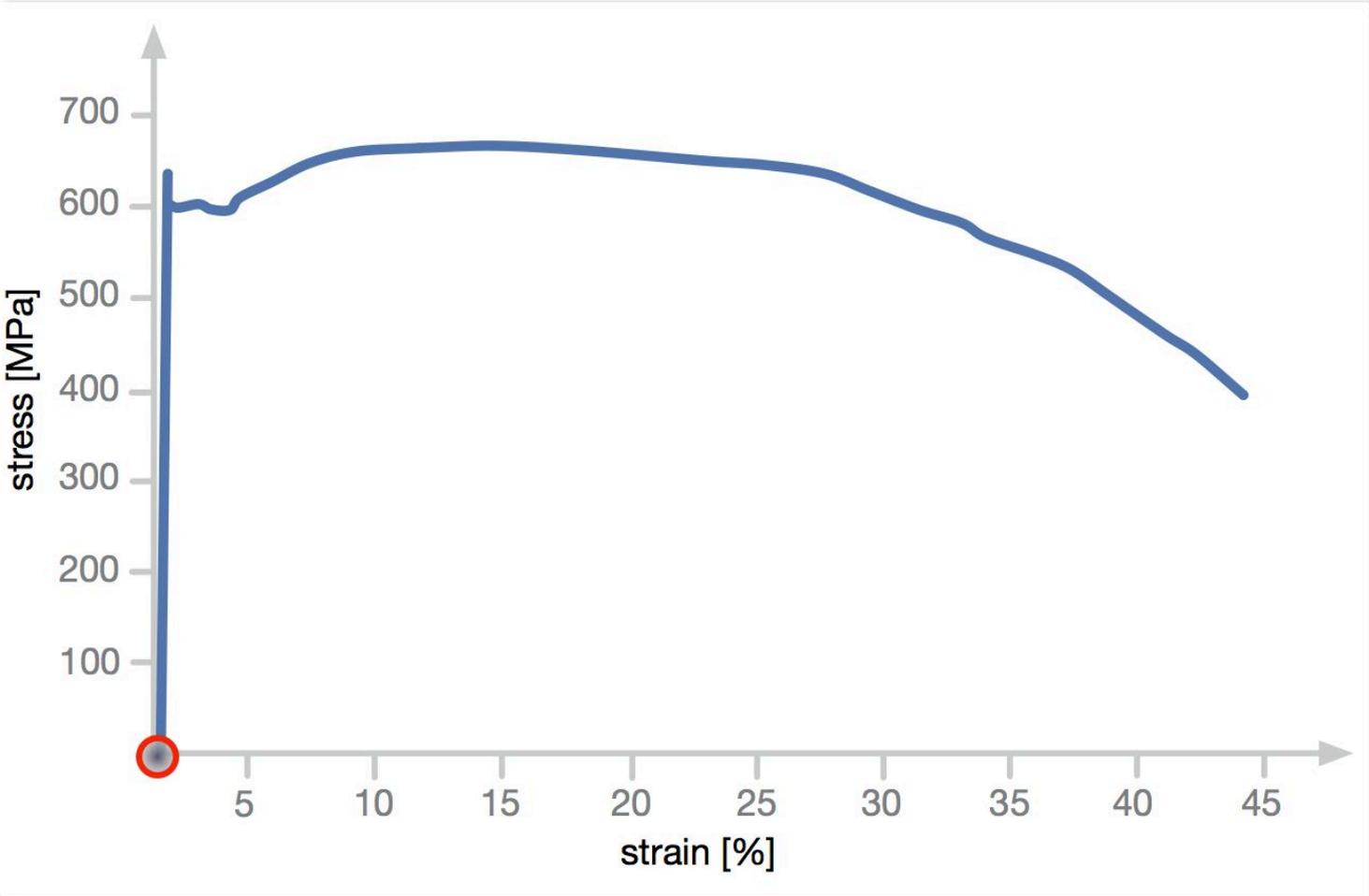
legami costitutivi

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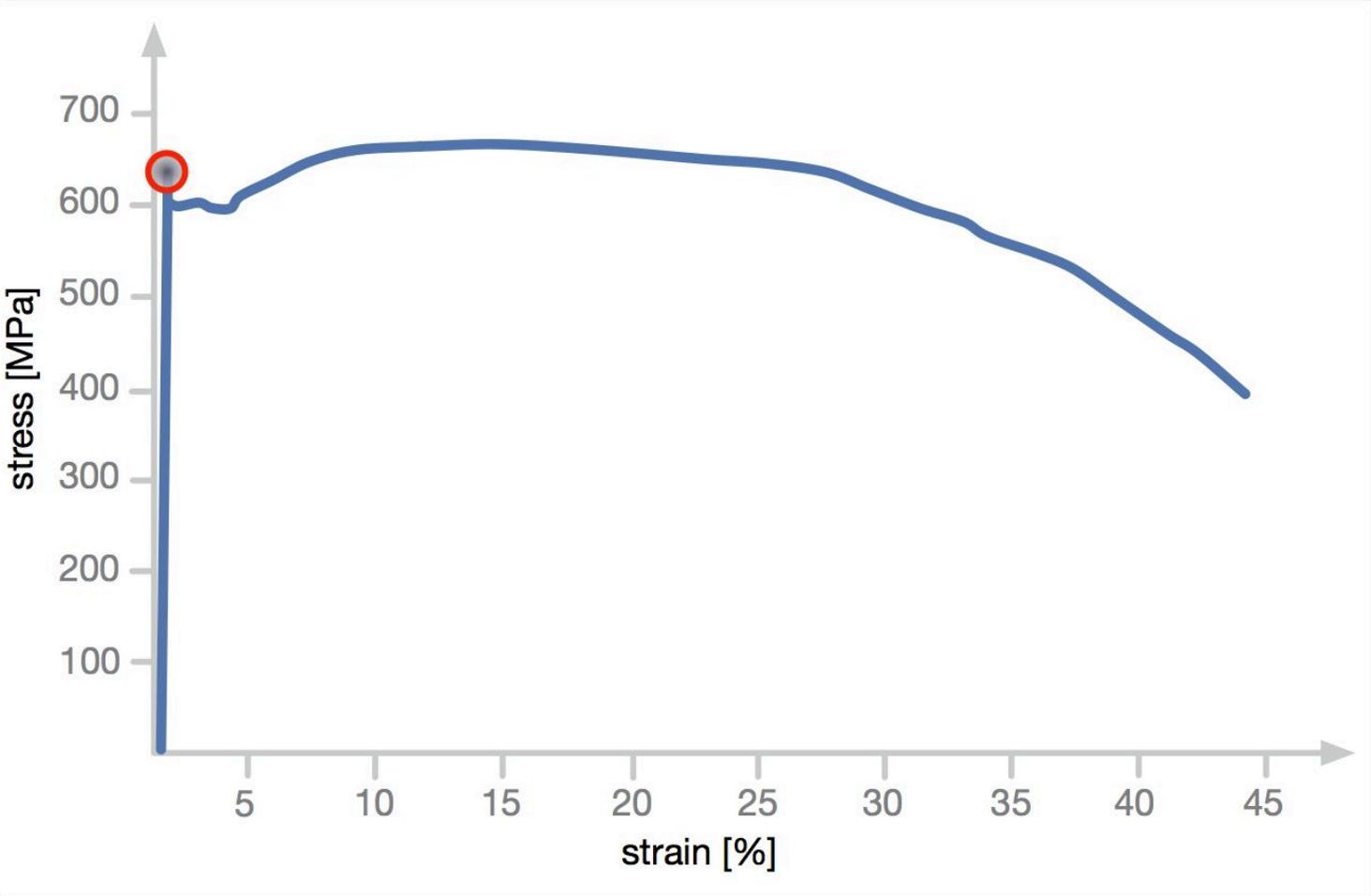
- modello di fibre estensibili



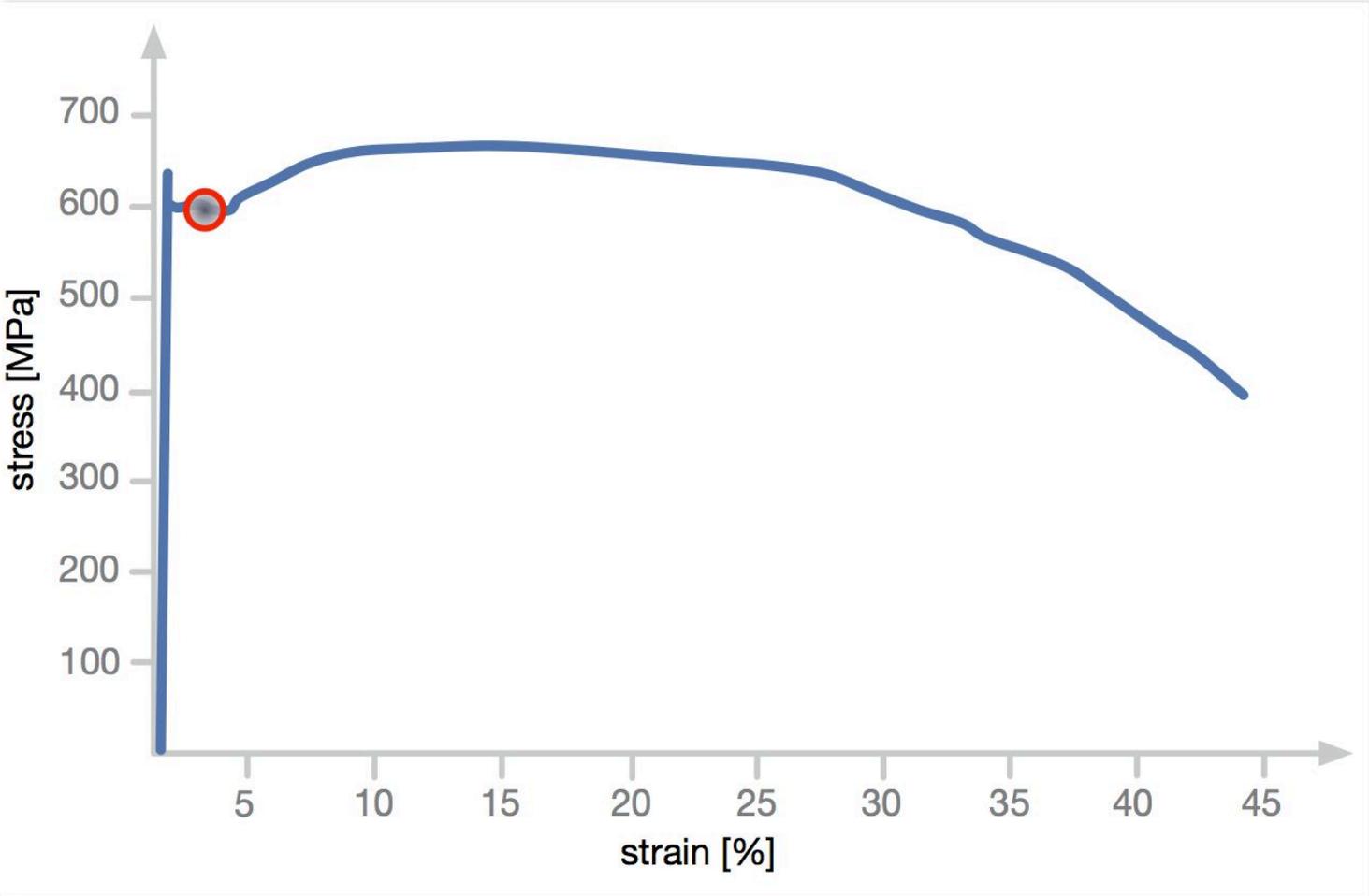
# test a trazione



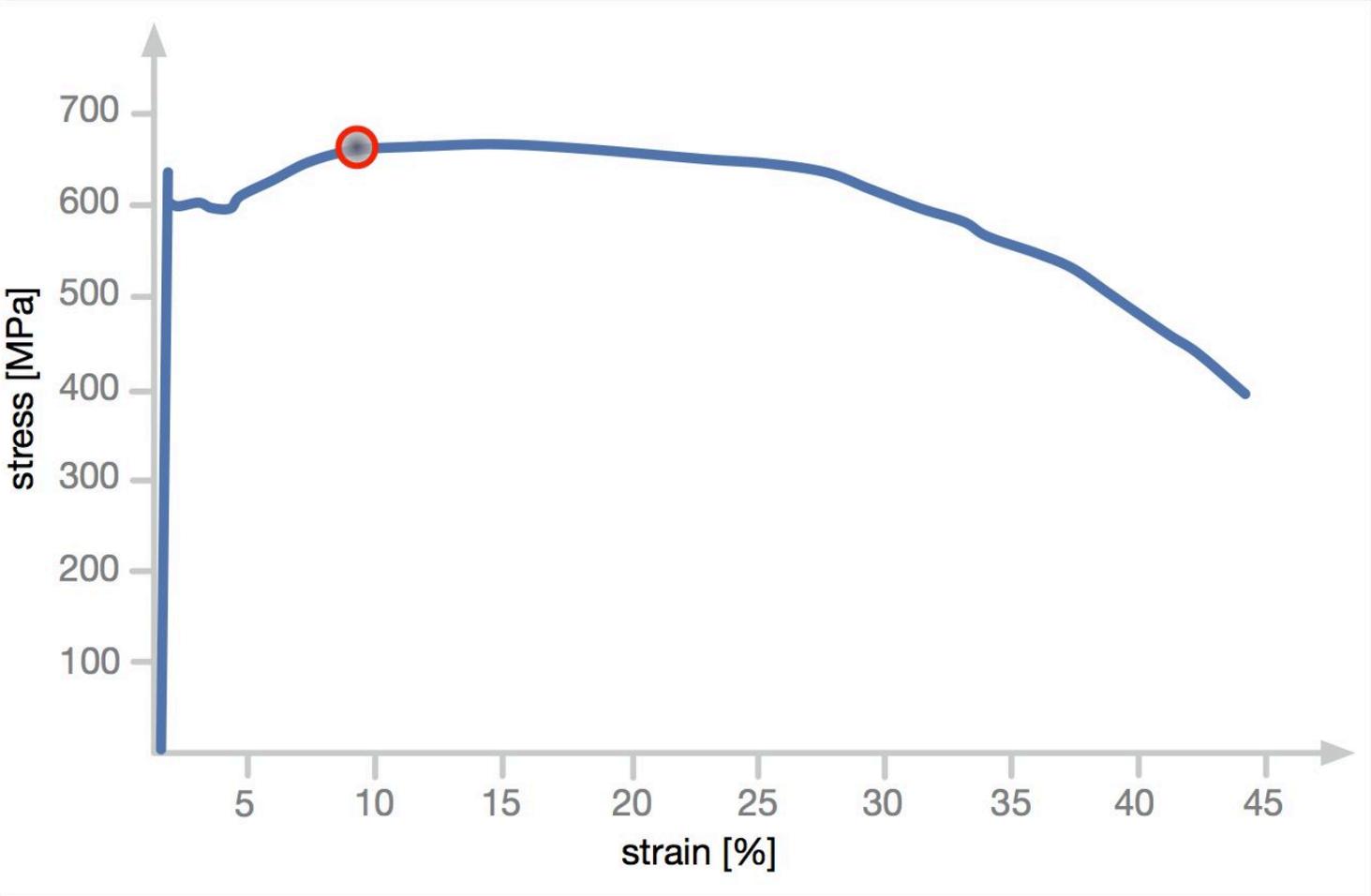
# test a trazione



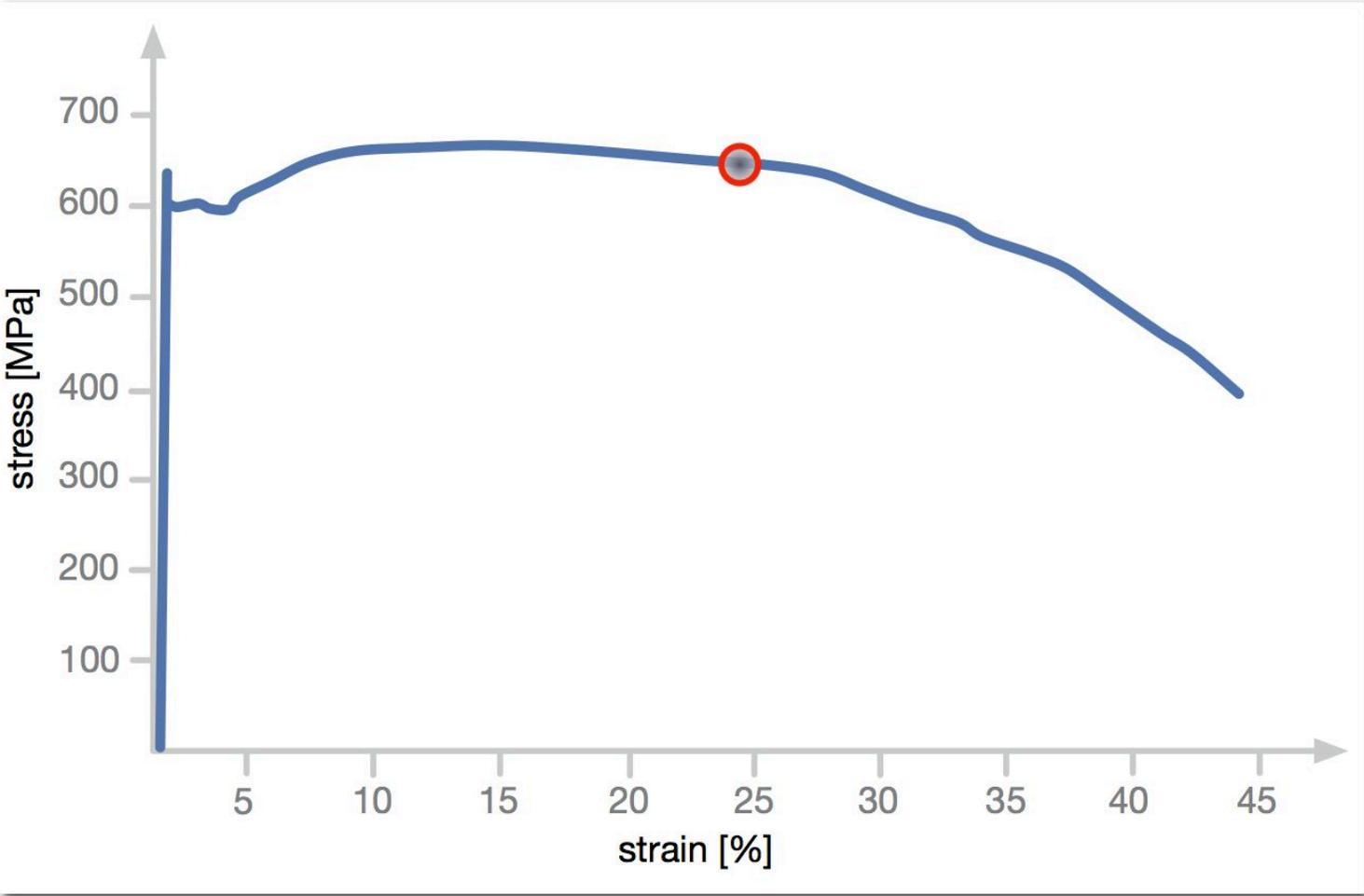
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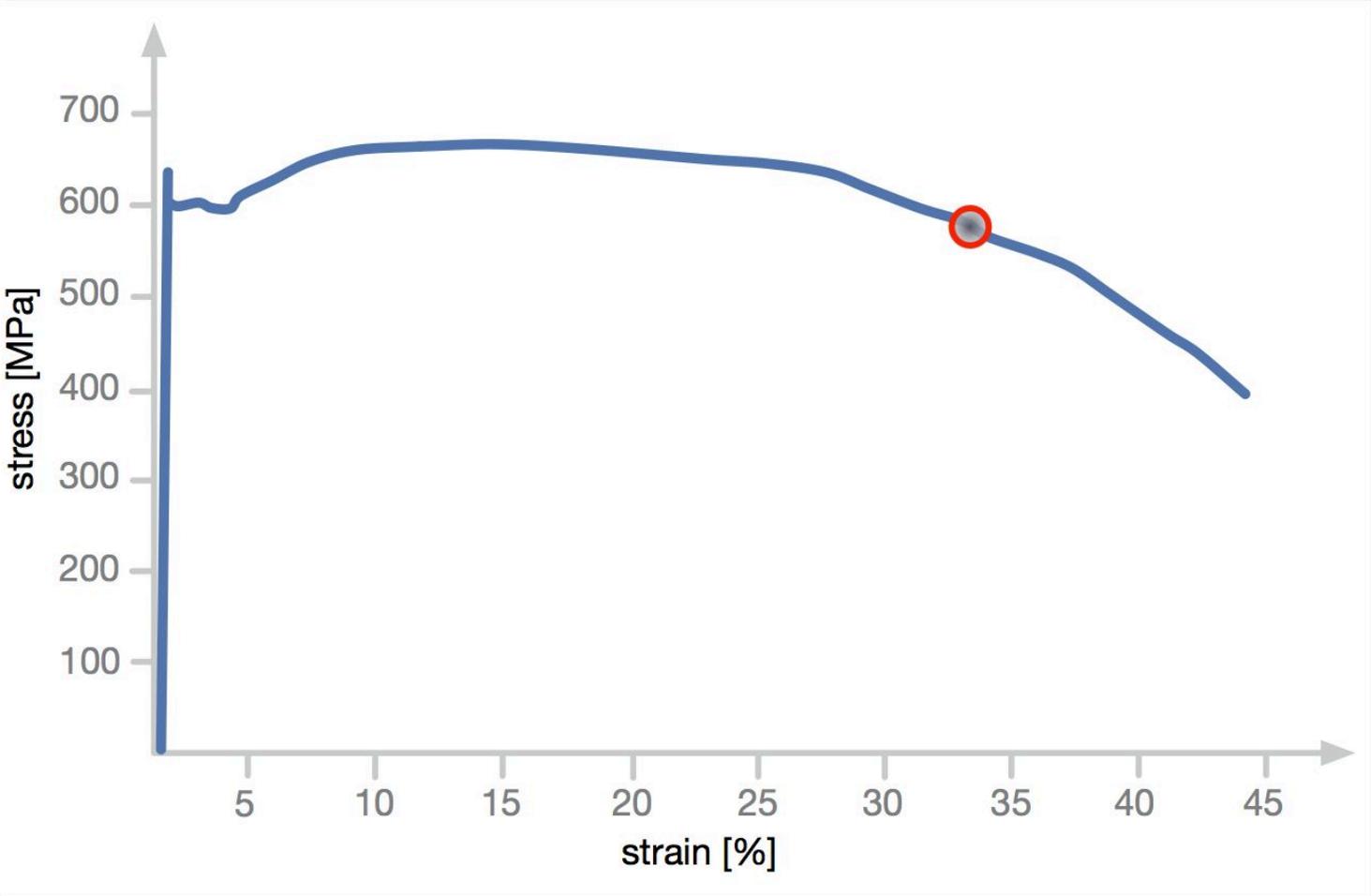
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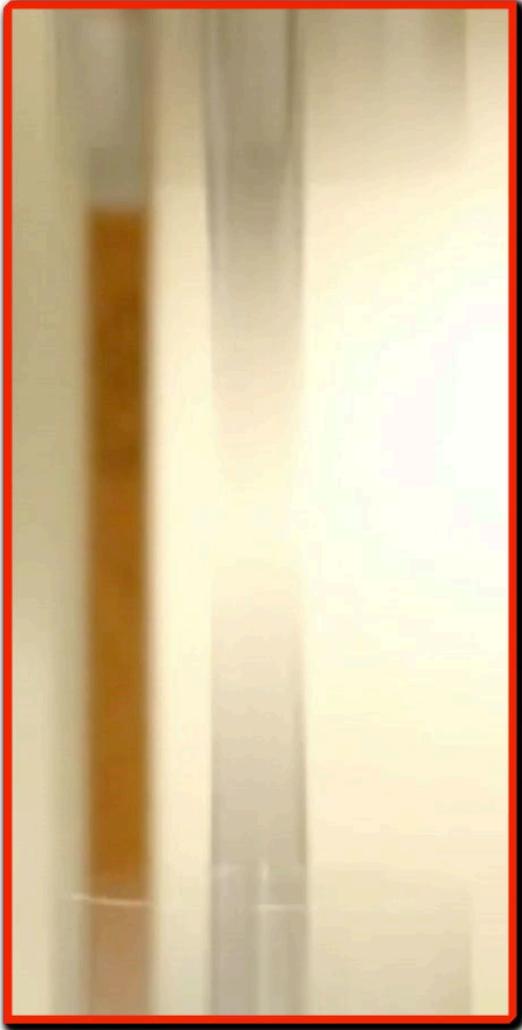
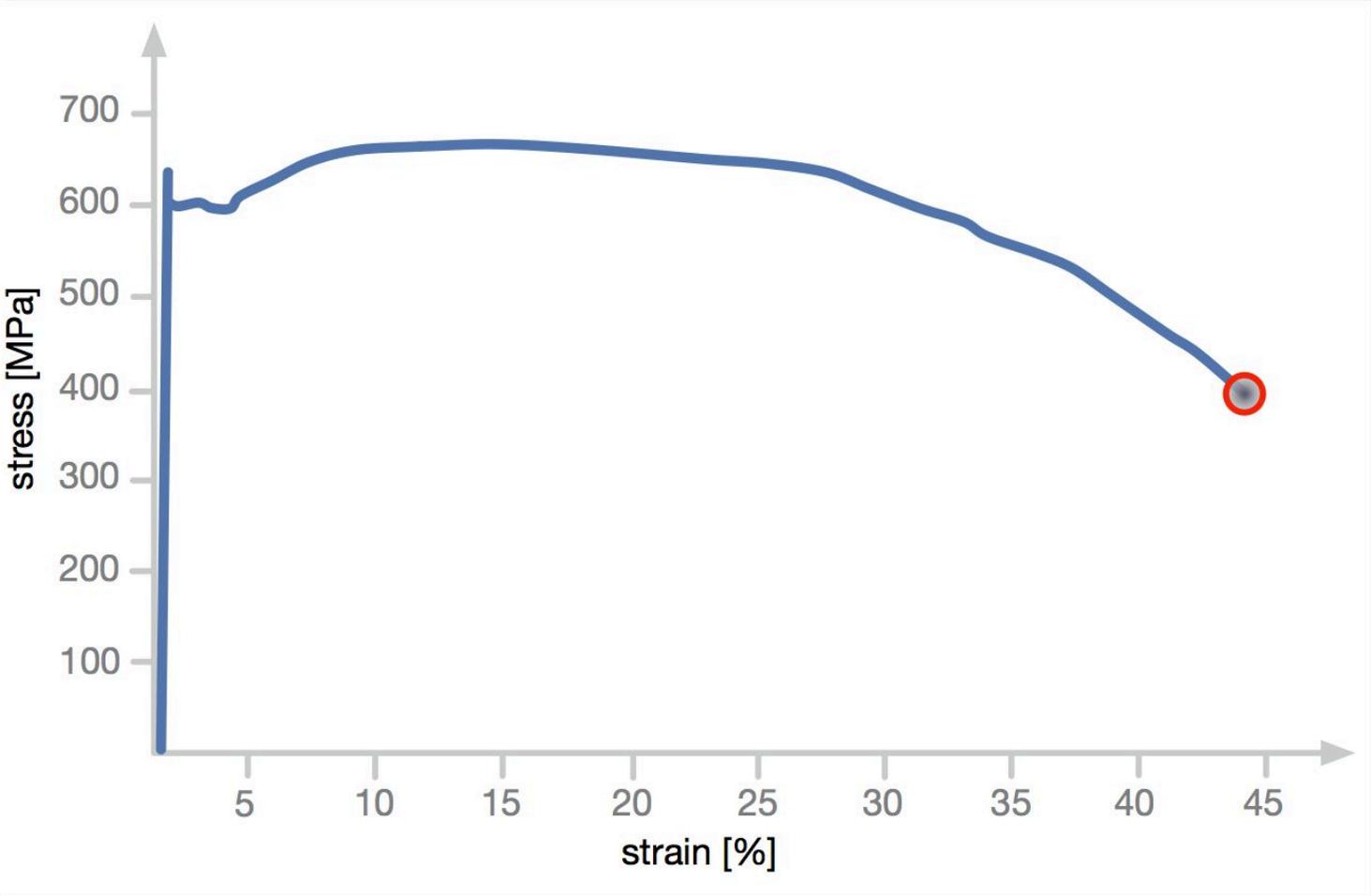
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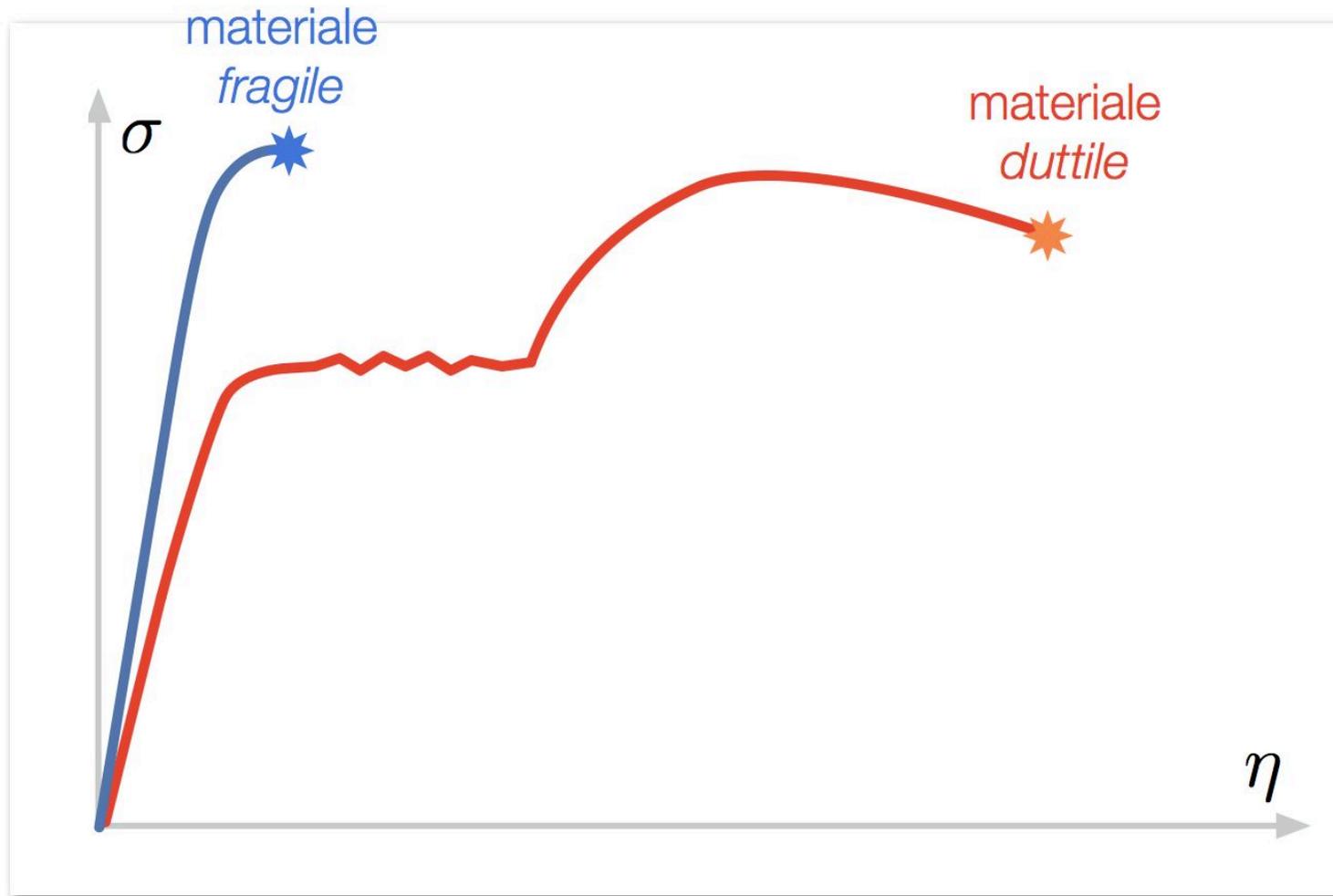


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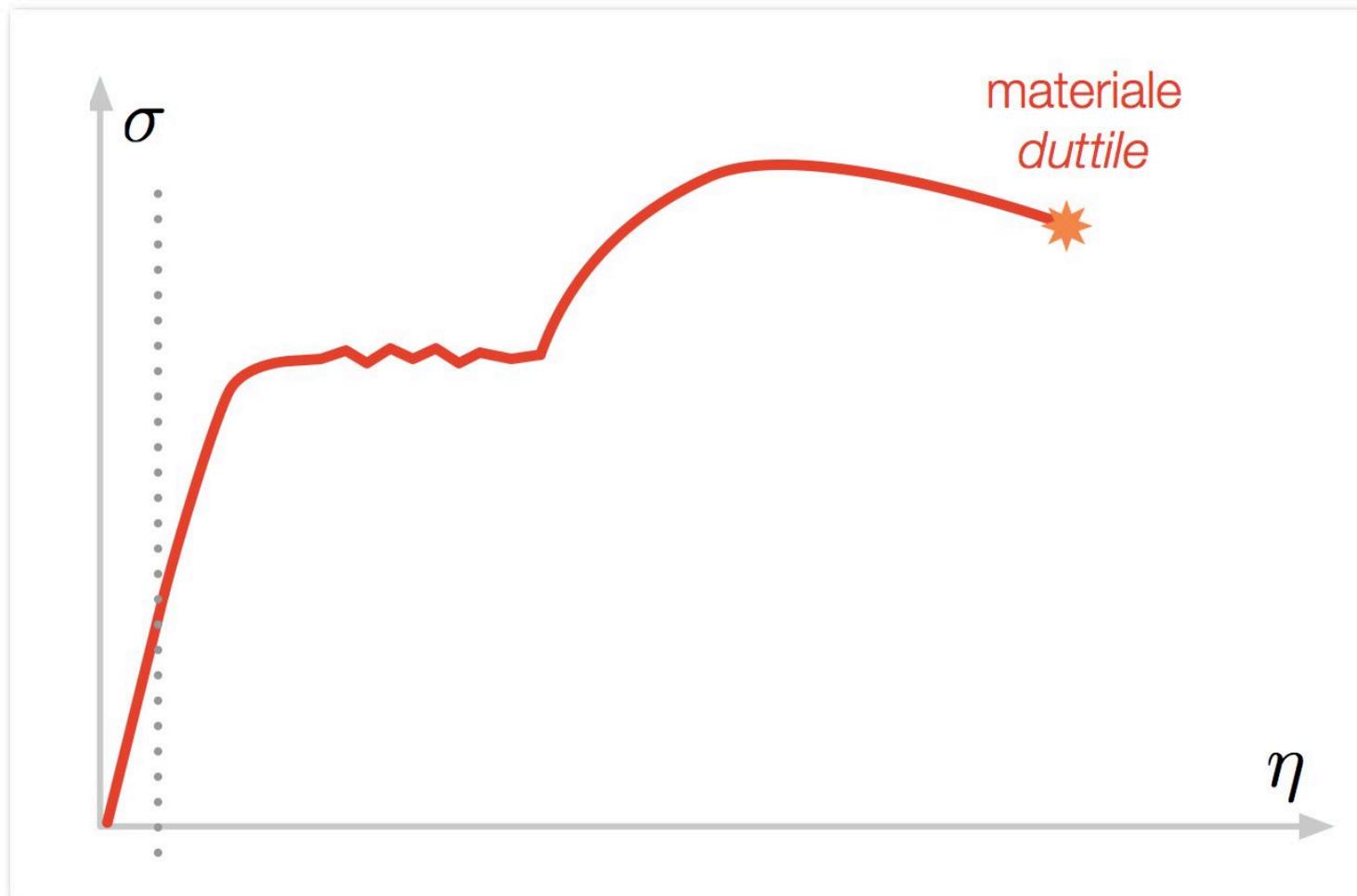
## modelli (reologici) di comportamento del materiale

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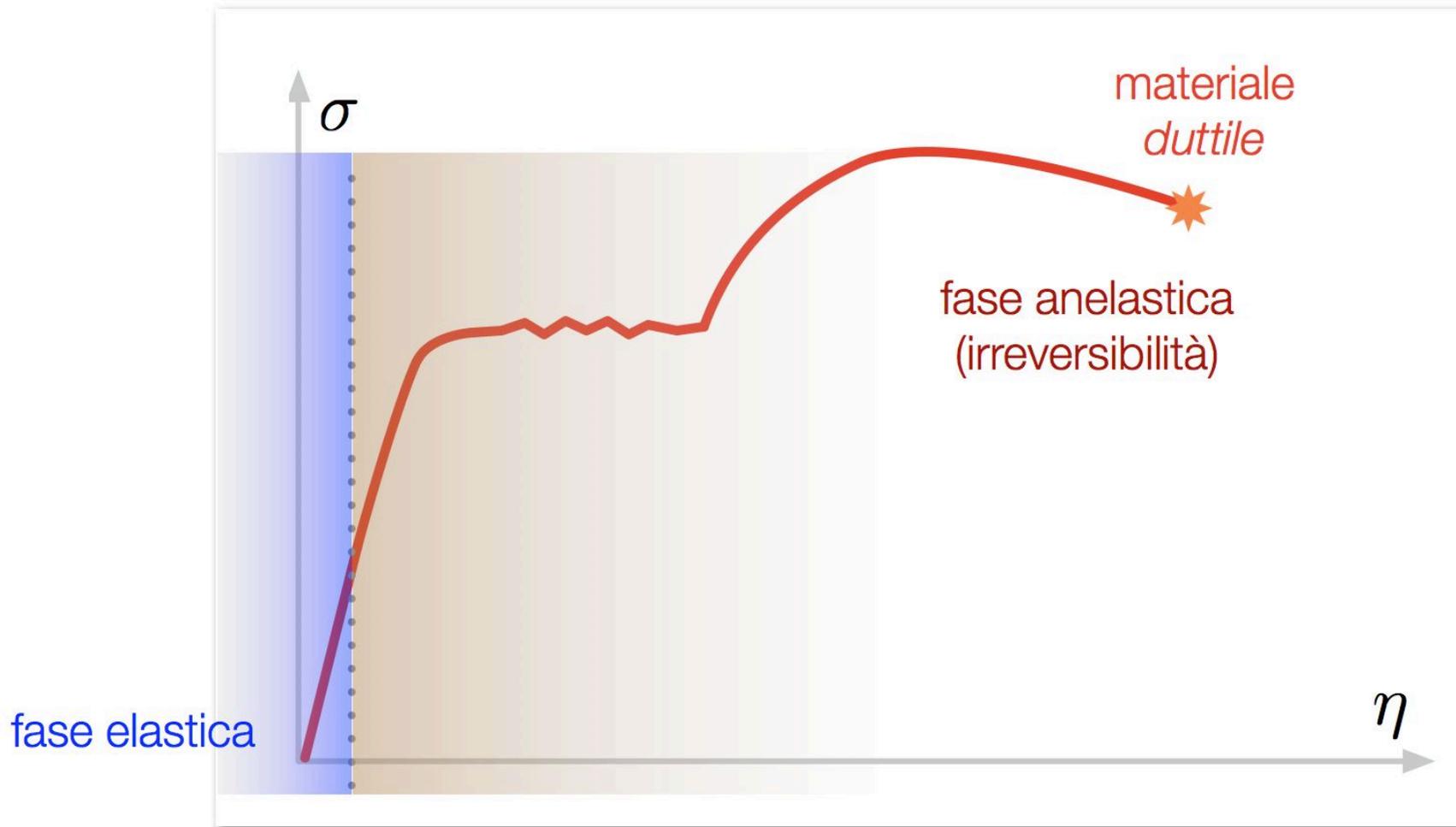


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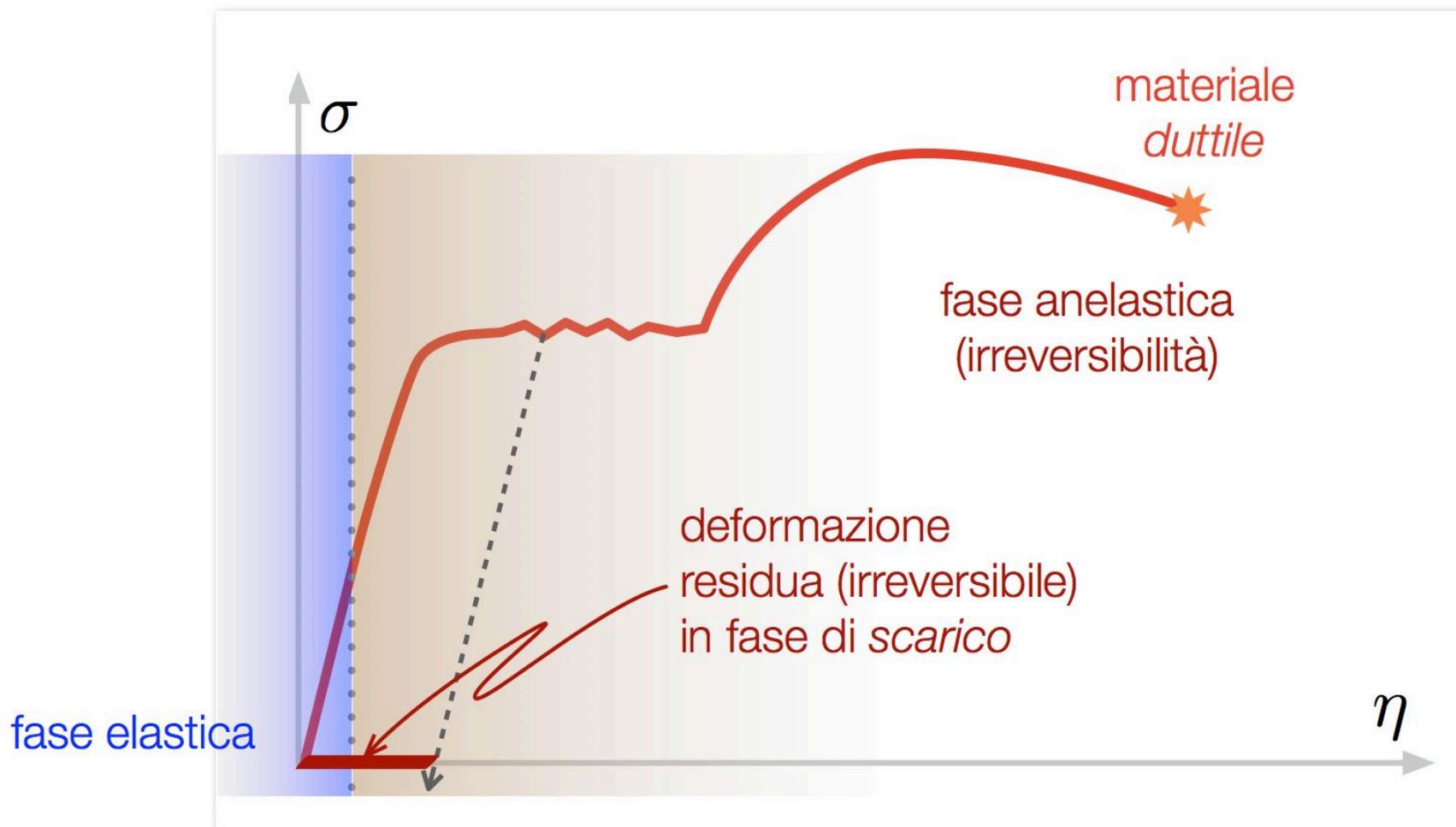
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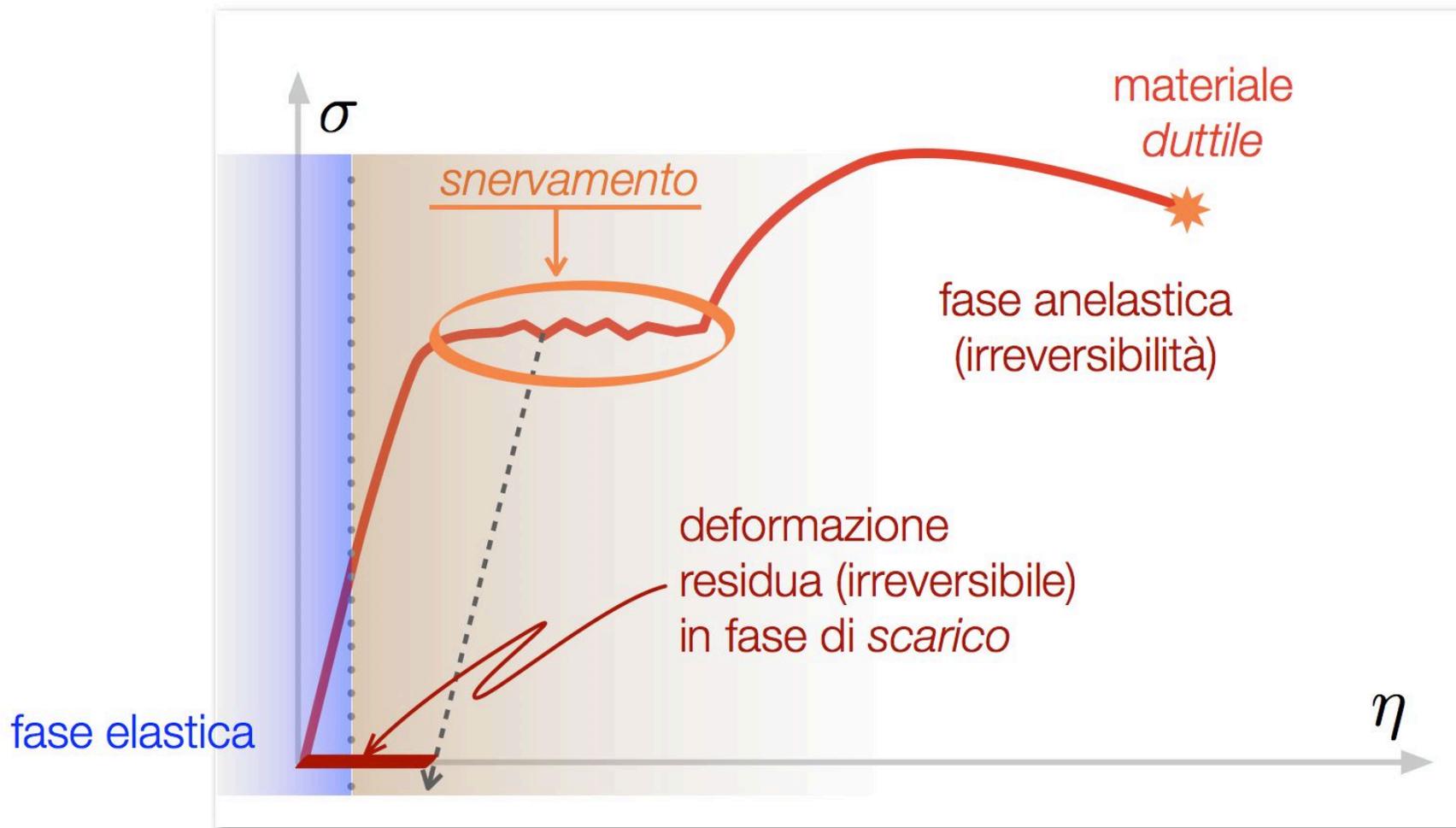
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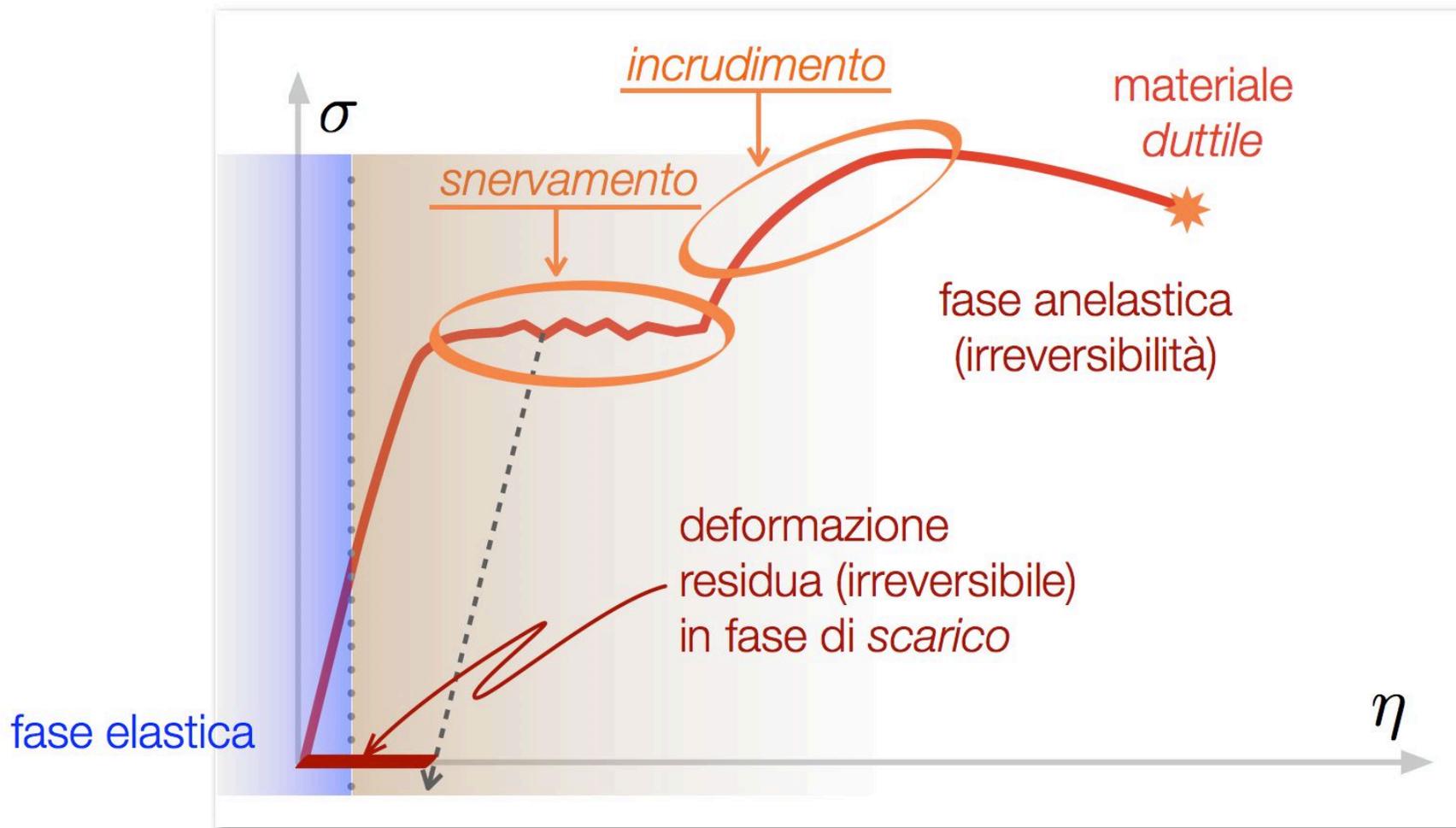
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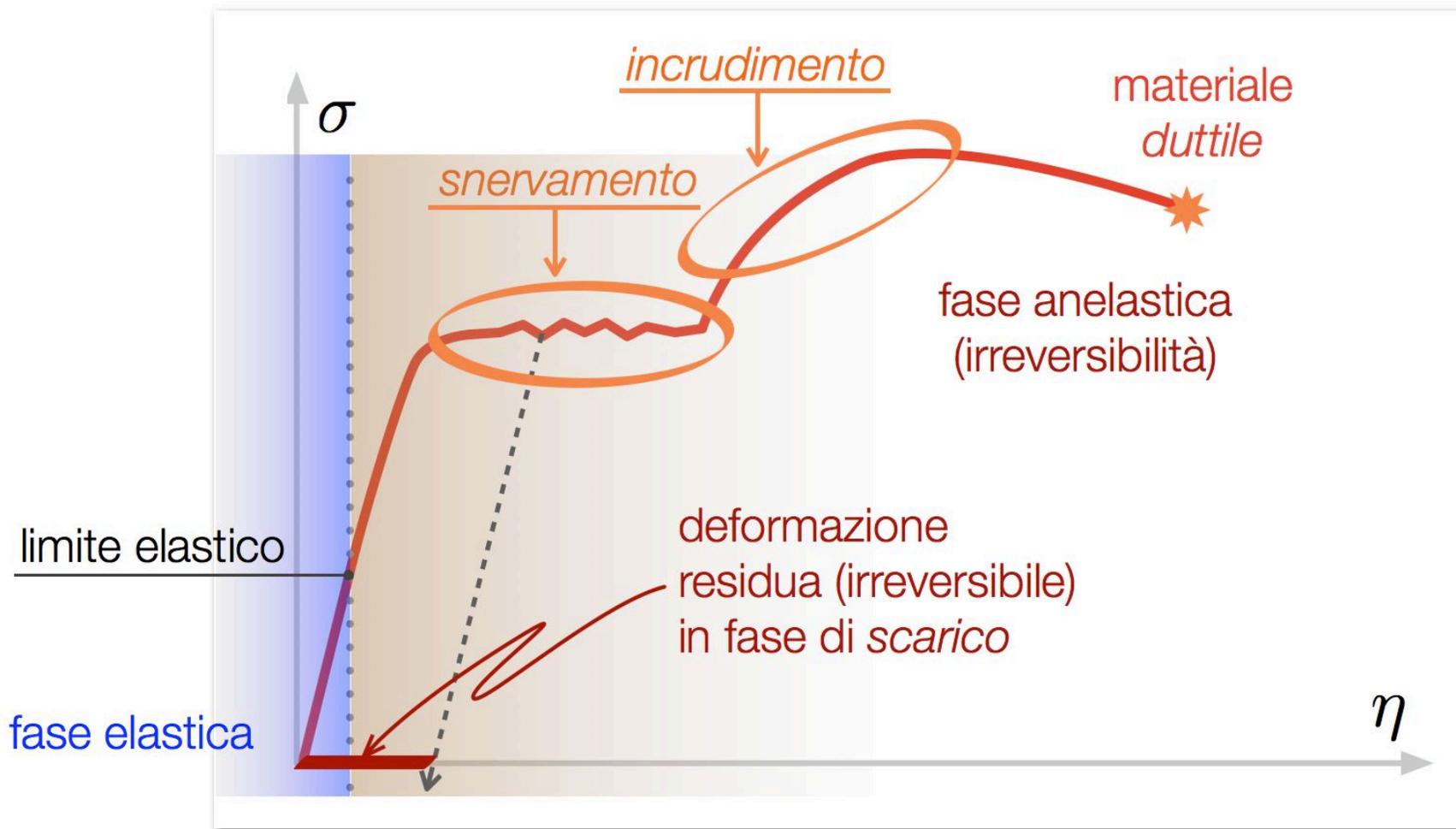
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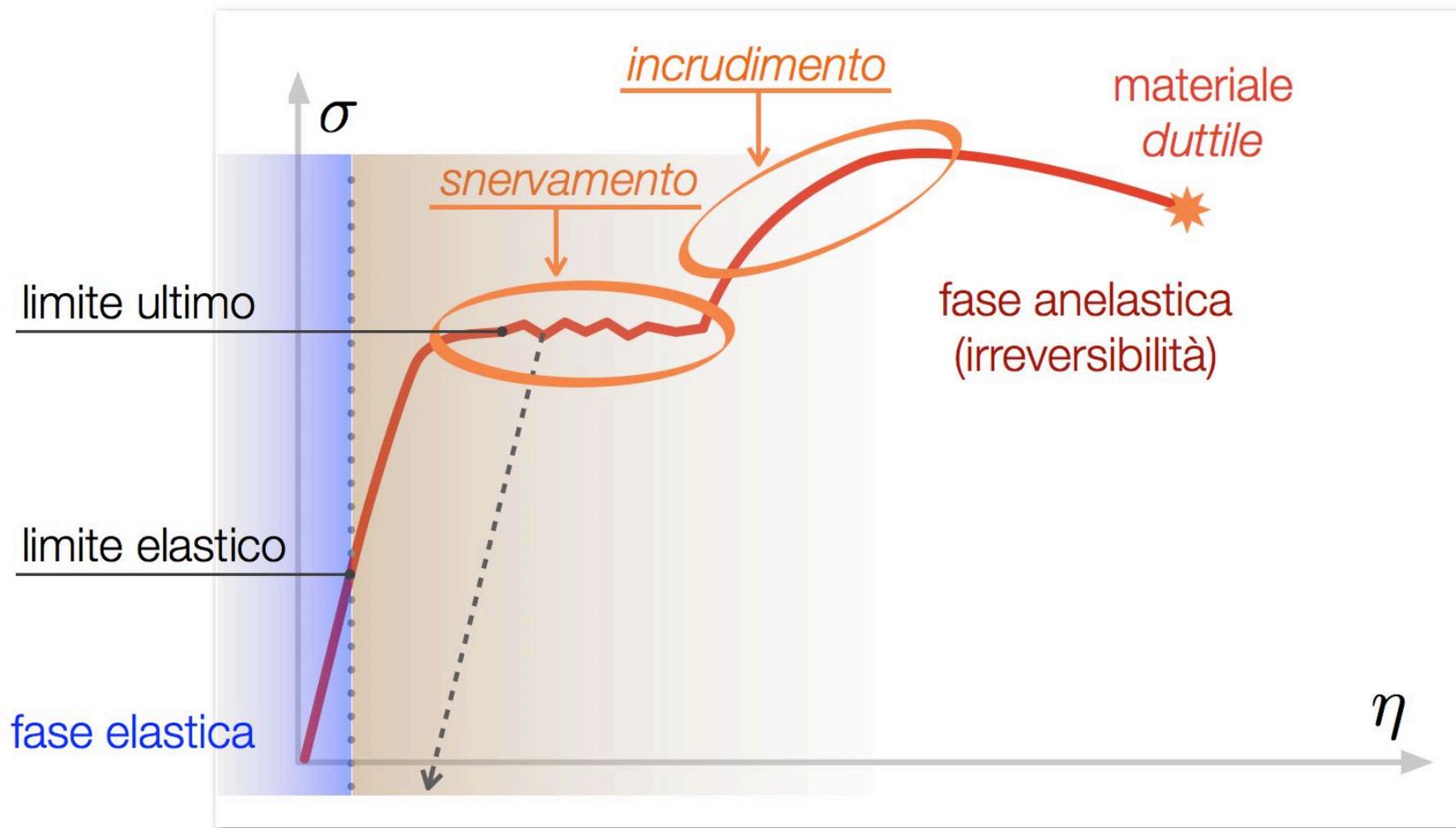
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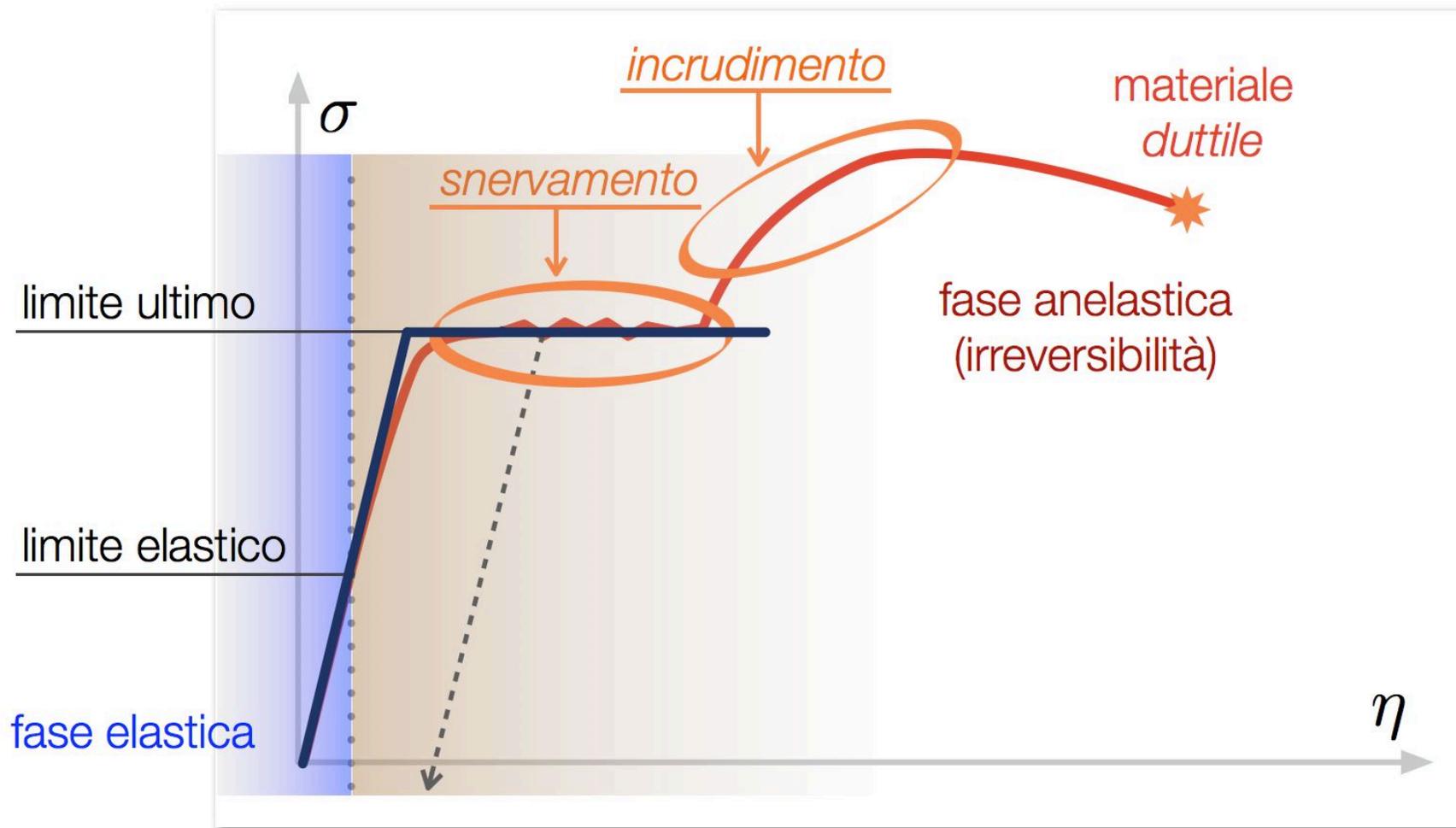
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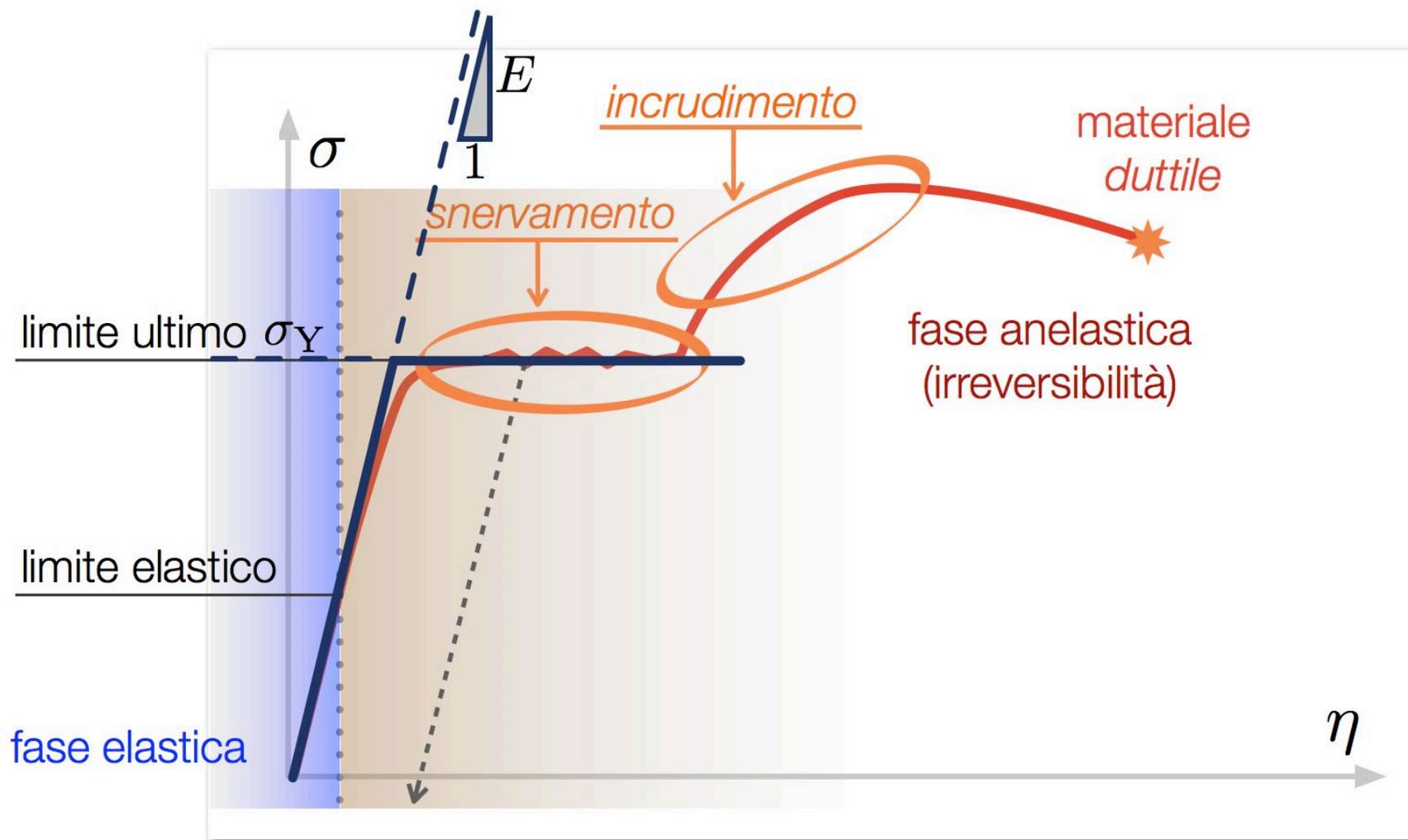
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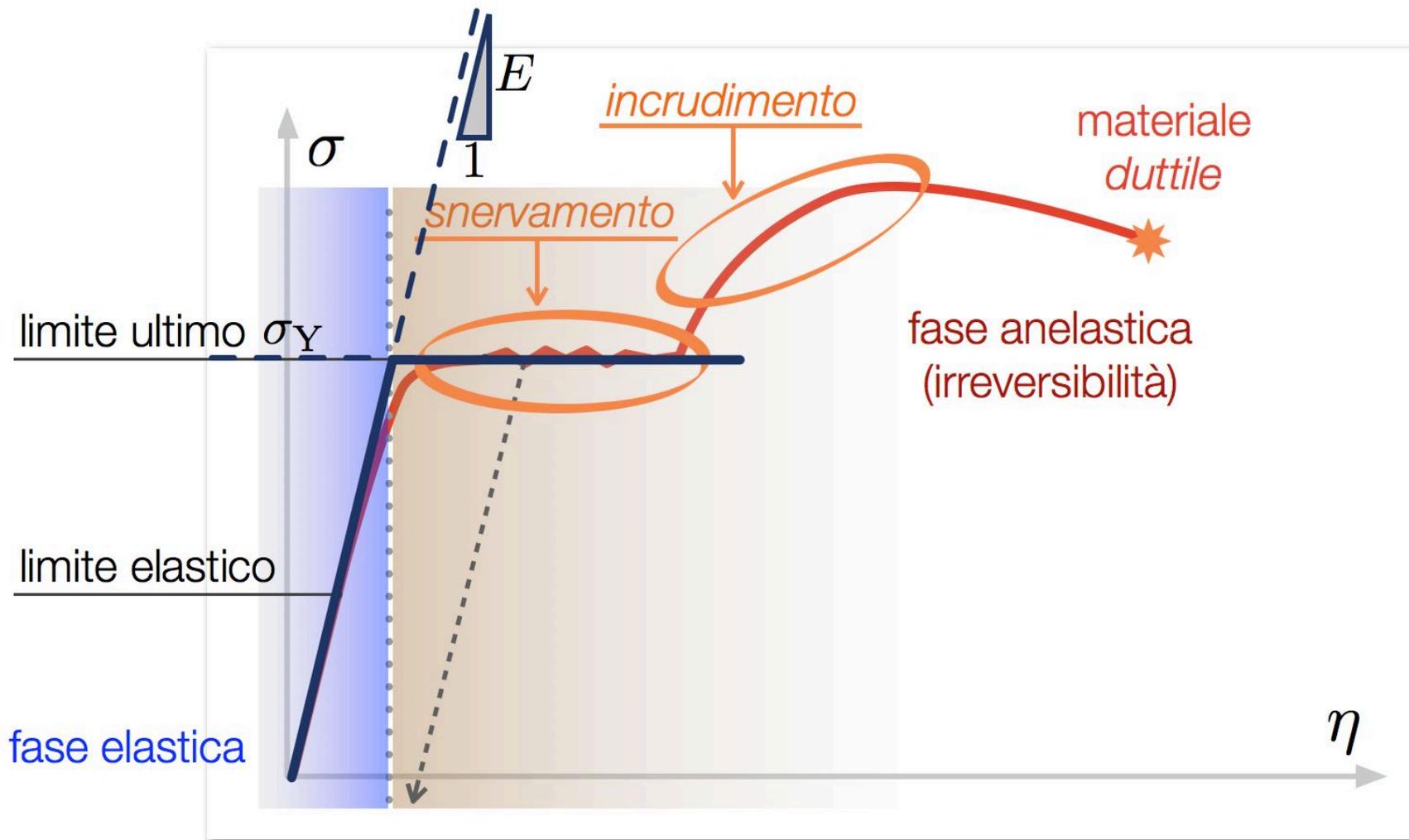
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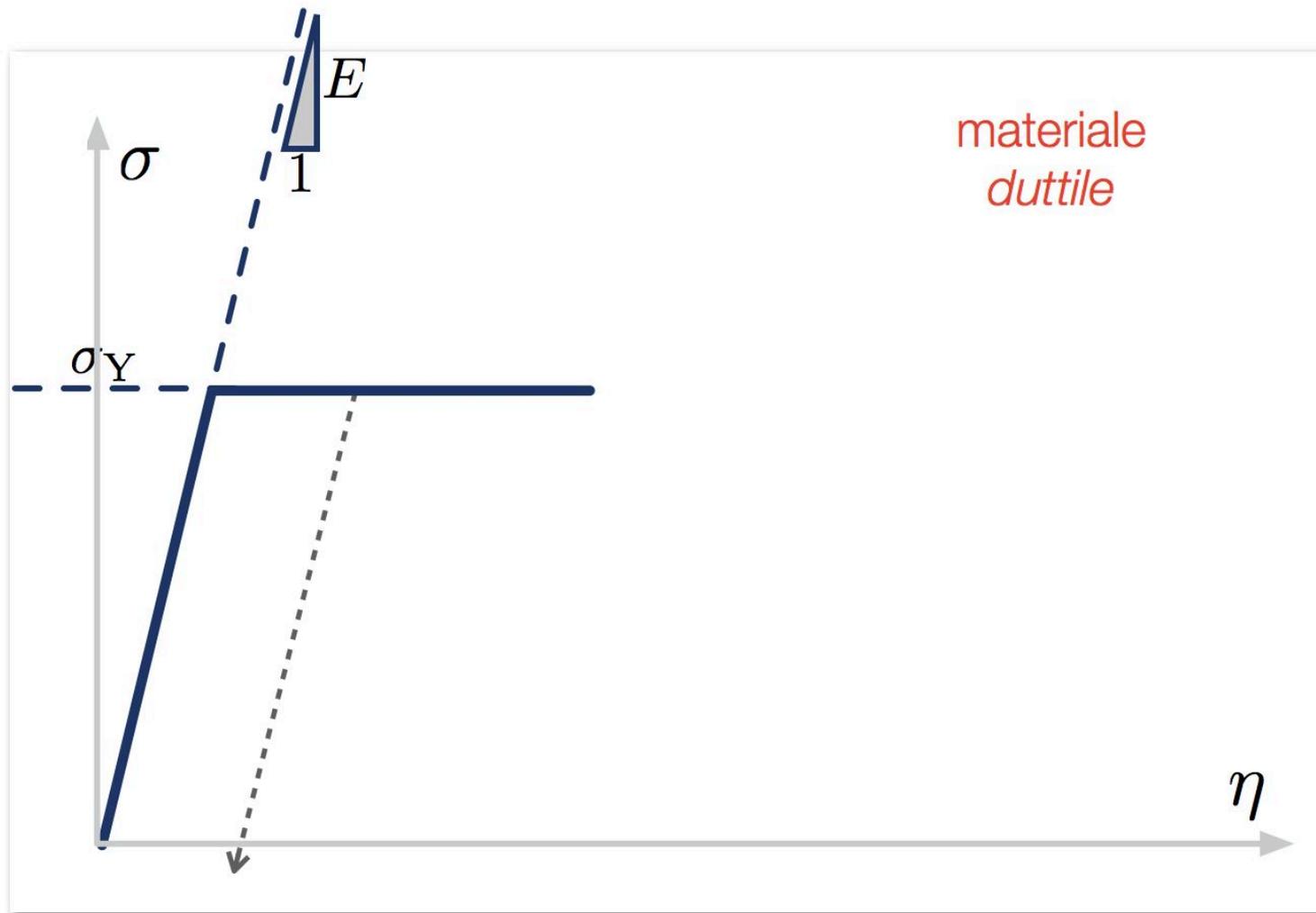
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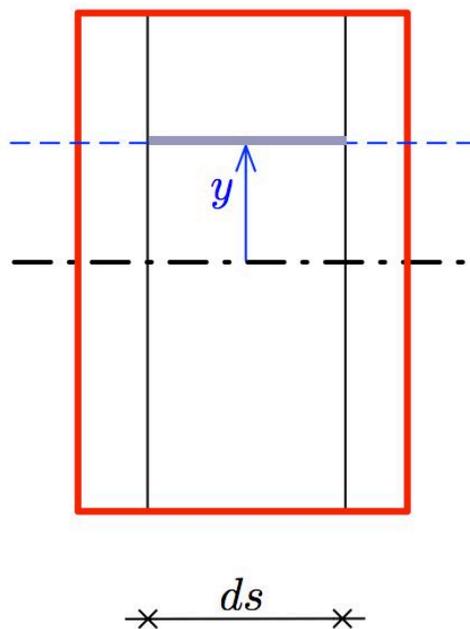
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legami costitutivi

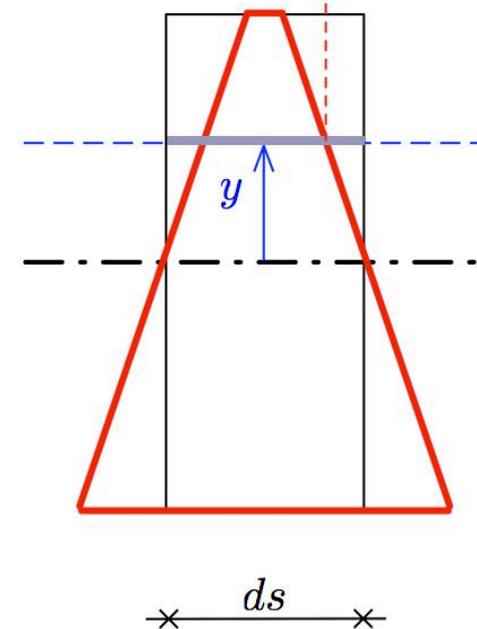
$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$



$$\eta ds/2 \equiv \varepsilon ds/2$$



$$\eta ds/2 \equiv \chi y ds/2$$

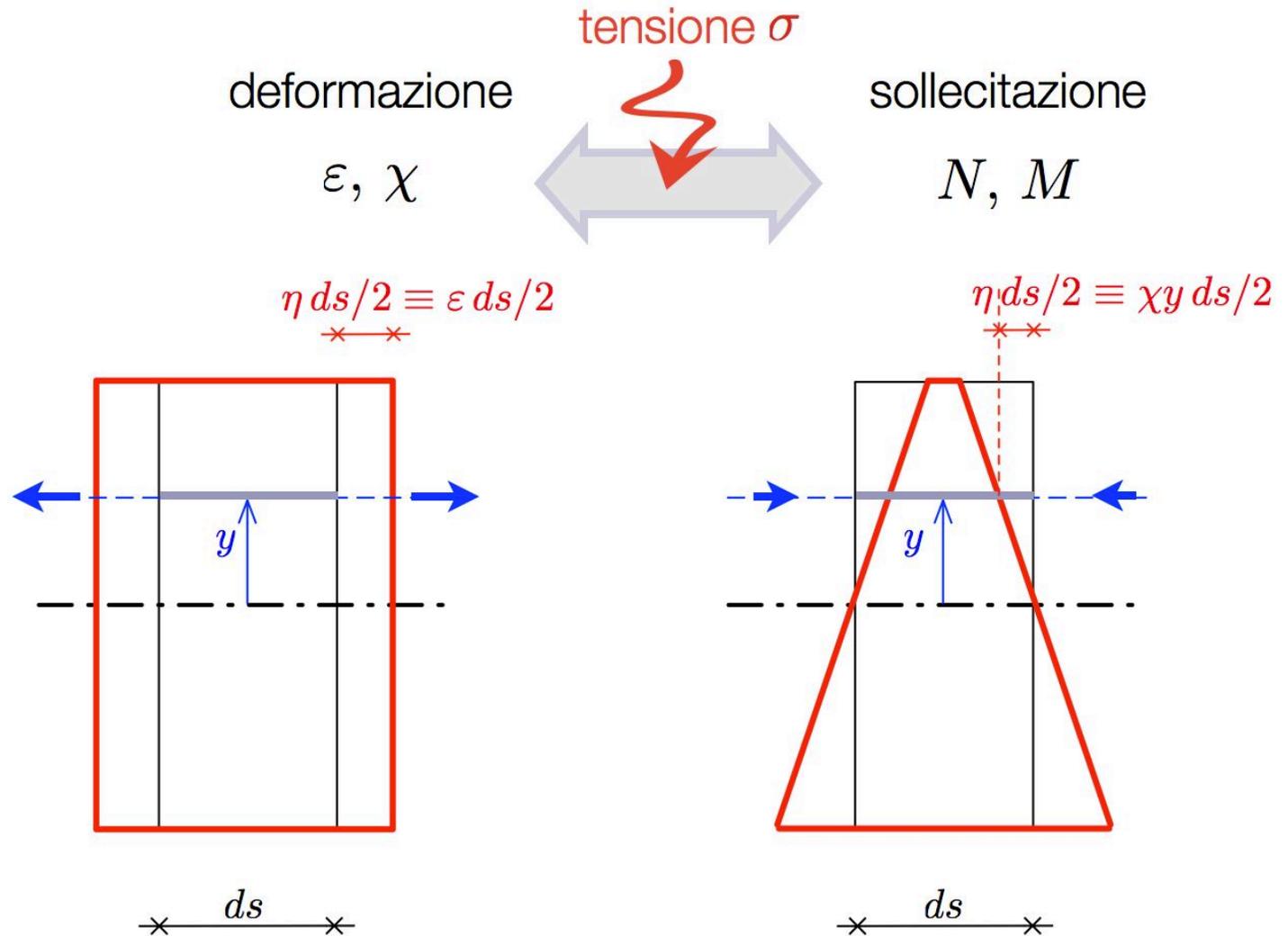


# stato tensionale per presso/tenso-flessione

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- modello di fibre estendibili
- legge di Hooke



# stato tensionale per presso/tenso-flessione

legami costitutivi

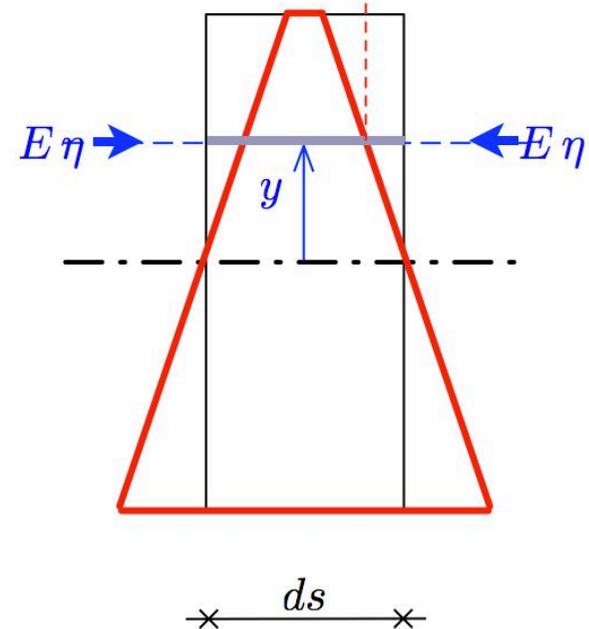
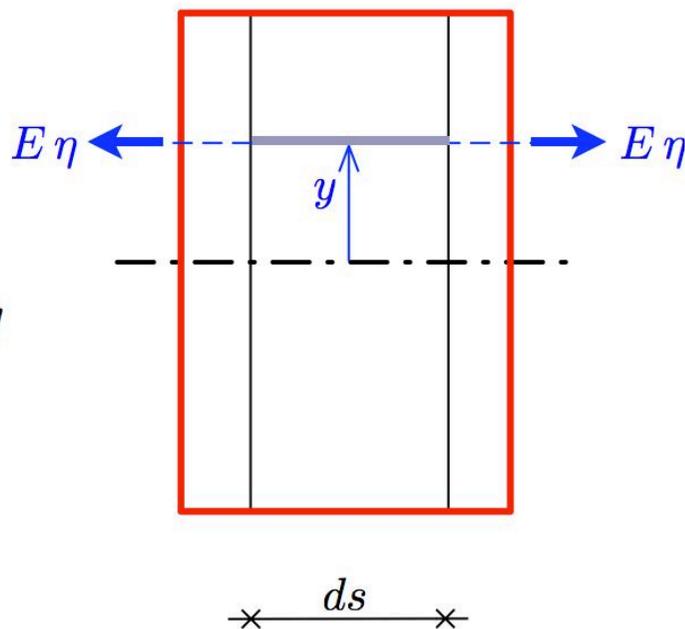
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- modello di fibre estendibili
- legge di Hooke  $\sigma = E\eta = E\varepsilon - E\chi y$



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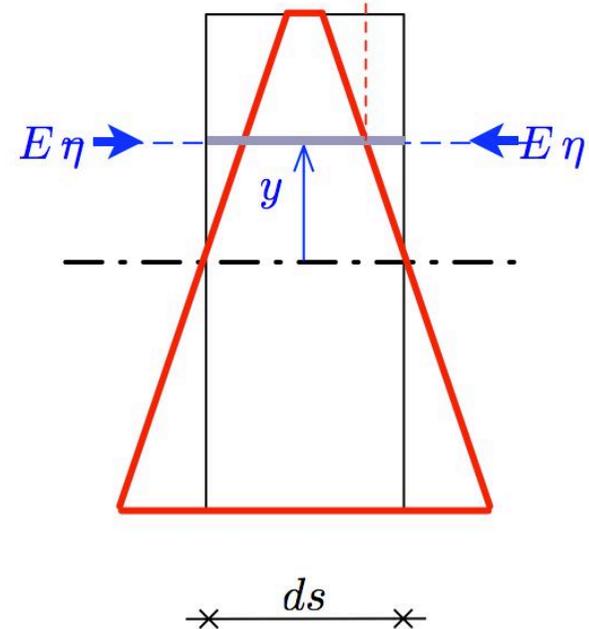
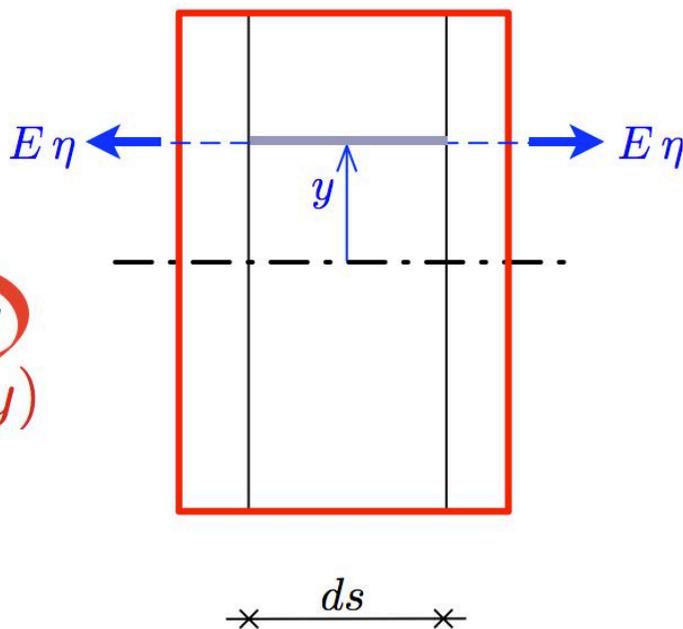
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- modello di fibre estendibili

- legge di Hooke

$$\sigma = E\eta = \underbrace{E\varepsilon}_{\sigma_n} - \underbrace{E\chi y}_{\sigma_m(y)}$$



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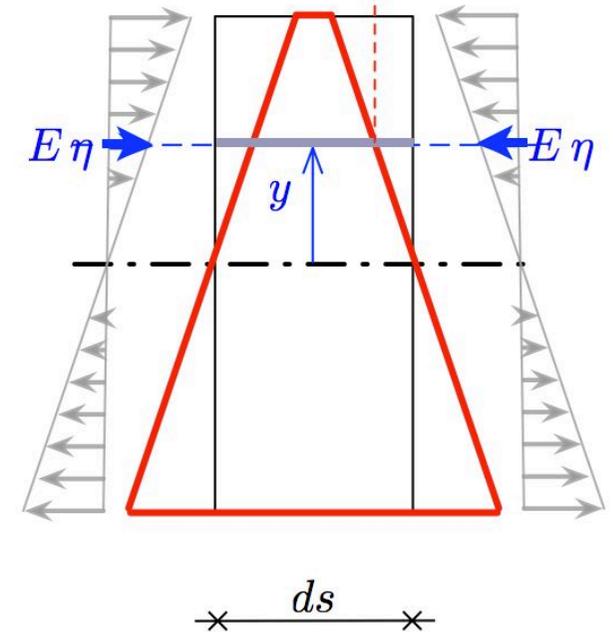
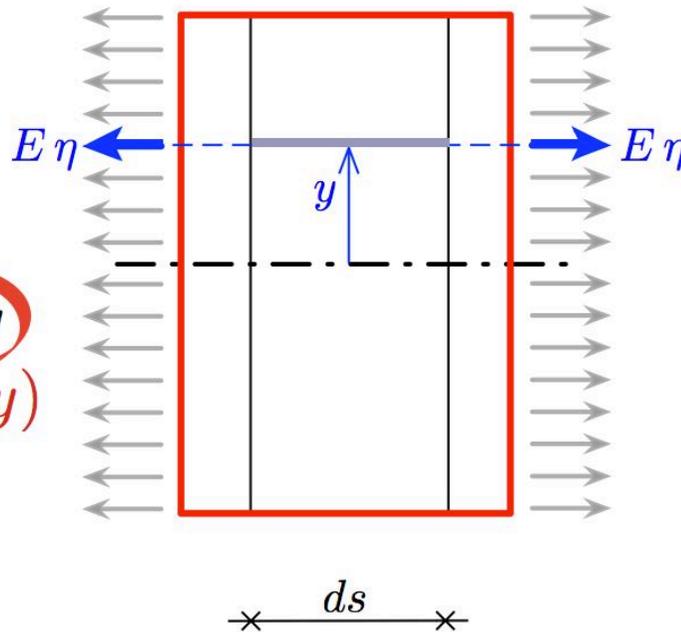
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## stato tensionale per presso/tenso-flessione

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legami costitutivi

$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$



- modello di fibre estendibili

$$\mathcal{P}_{\text{int}} = \int_{\ell} N \dot{\varepsilon} + M \dot{\chi} \equiv \int_{\ell} \int_A \sigma \dot{\eta}$$

- legge di Hooke

$$\sigma = E\eta = E\varepsilon - E\chi y$$

- equivalenza di potenza (assi baricentrici)

## stato tensionale per presso/tenso-flessione

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legami costitutivi

$$\begin{cases} N[s] = EA \varepsilon[s] \\ M[s] = EJ \chi[s] \end{cases}$$



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## stato tensionale per presso/tenso-flessione

legami costitutivi

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- modello di fibre estendibili

- legge di Hooke

$$\sigma = E\eta = \underbrace{E\varepsilon}_{\sigma_n} - \underbrace{E\chi y}_{\sigma_m(y)} = \frac{N}{A} - \frac{M}{J}y$$

- equivalenza di potenza (assi baricentrici)

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$$\equiv EA \varepsilon \dot{\varepsilon} + EJ \chi \dot{\chi}$$

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# stato tensionale per presso/tenso-flessione

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- legge di Hooke

$$\sigma = E\eta = E\varepsilon - E\chi y = \underbrace{\frac{N}{A}}_{\sigma_n} - \underbrace{\frac{M}{J}y}_{\sigma_m(y)}$$

- equivalenza di potenza (assi baricentrici)

$$\mathcal{P}_{\text{int}} = \int_{\ell} N\dot{\varepsilon} + M\dot{\chi} \equiv \int_{\ell} \int_A \sigma\dot{\eta}$$

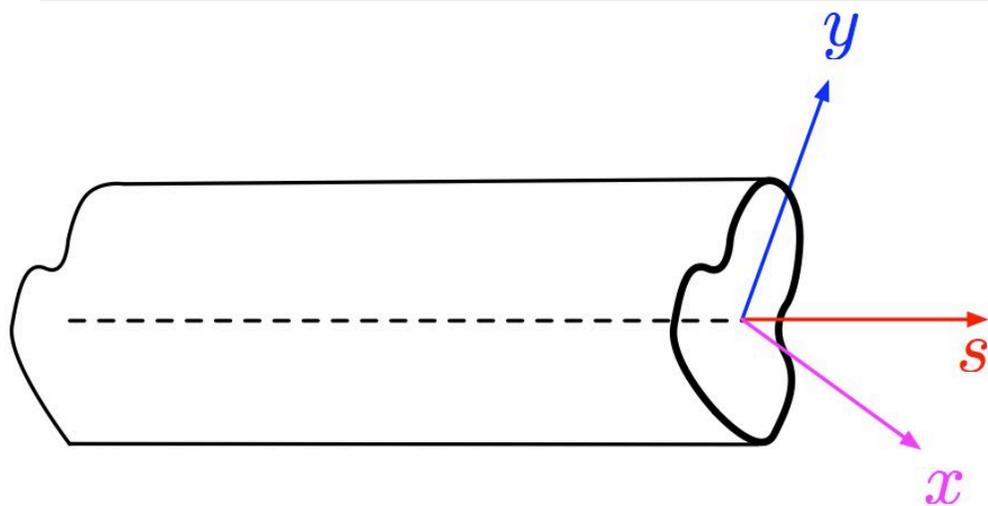
$$N\dot{\varepsilon} + M\dot{\chi} \equiv \int_A E\varepsilon\dot{\varepsilon} + \int_A E\chi y^2 \dot{\chi}$$

$$\equiv EA\varepsilon\dot{\varepsilon} + EJ\chi\dot{\chi}$$

$$\equiv \int_A \sigma_n \dot{\varepsilon} + \int_A \sigma_m(y) y \dot{\chi}$$

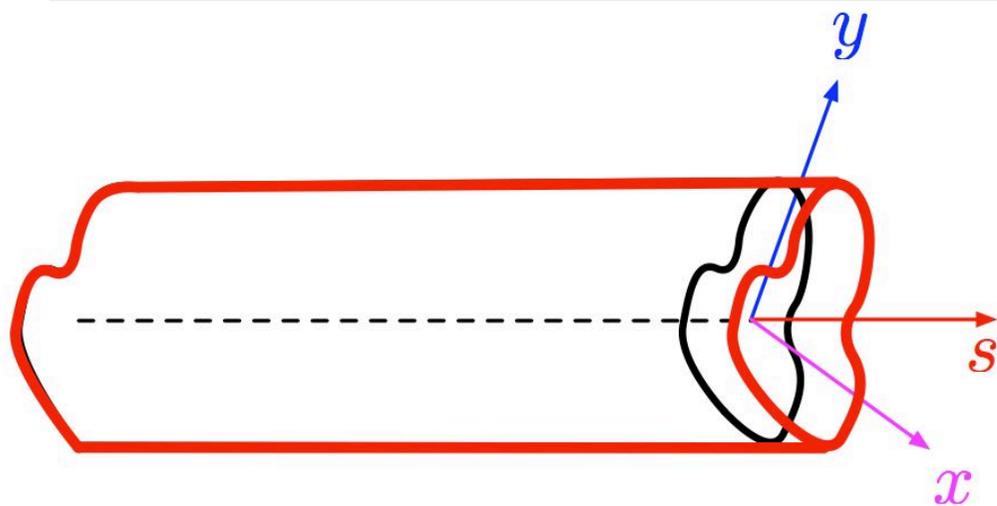
stato tensionale per presso/tenso-flessione

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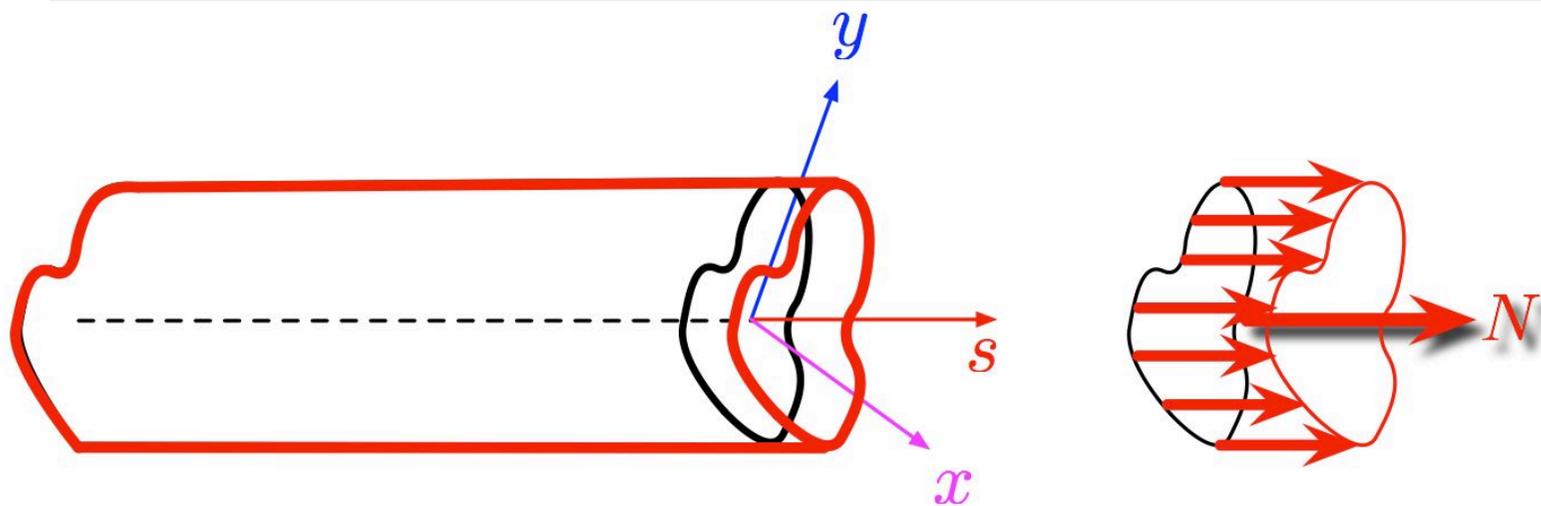
stato tensionale per presso/tenso-flessione

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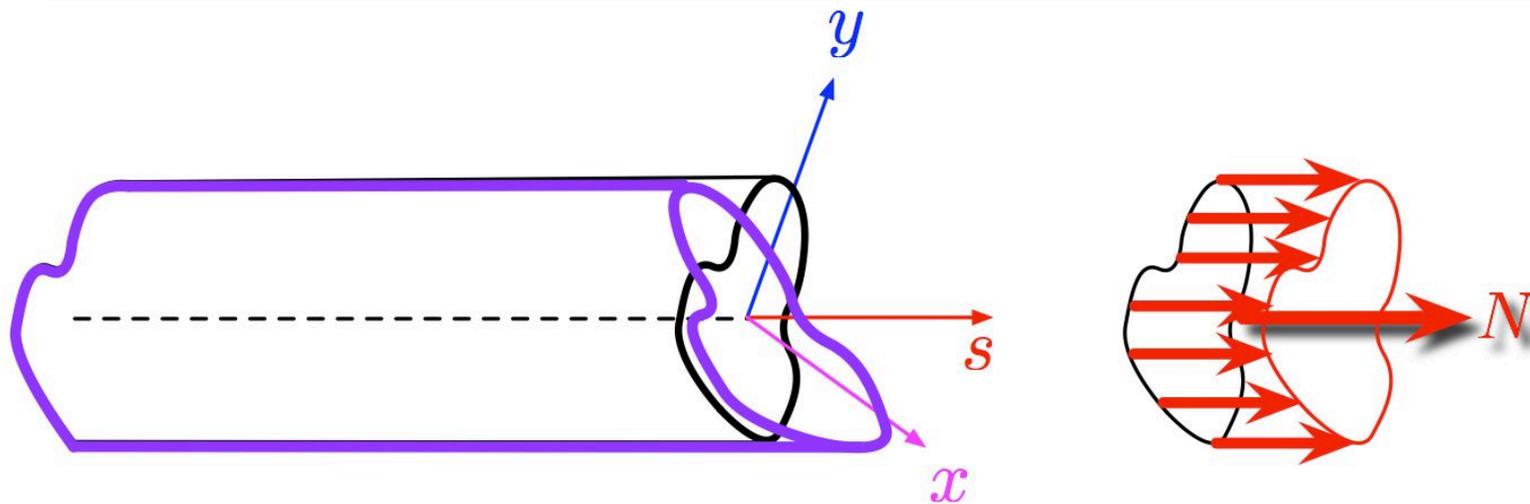
# stato tensionale per presso/tenso-flessione

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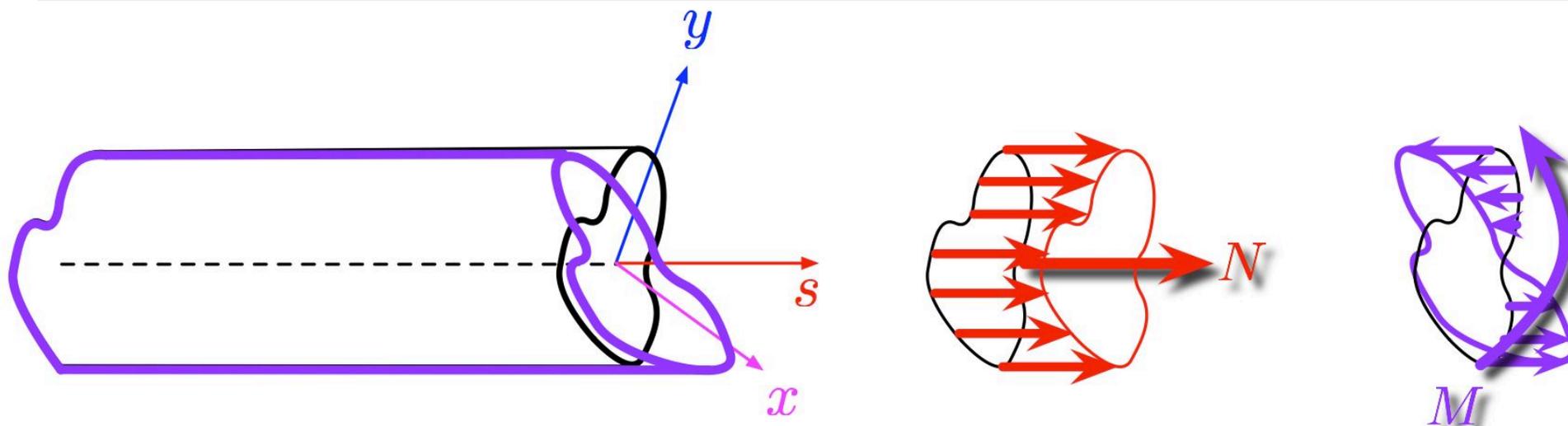
# stato tensionale per presso/tenso-flessione

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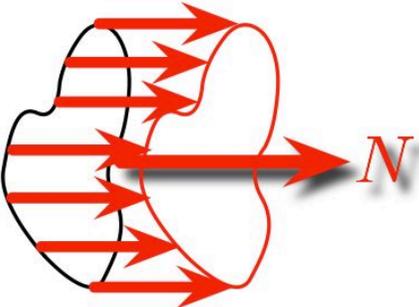
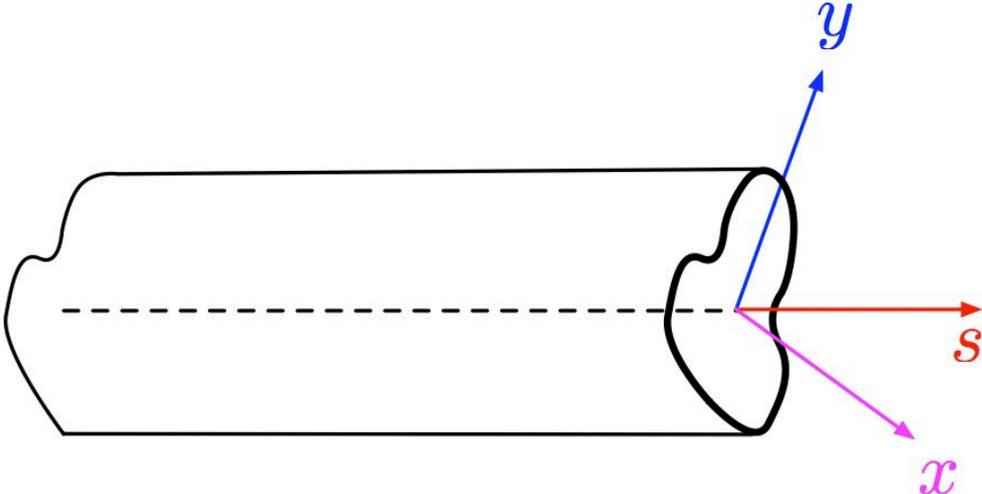


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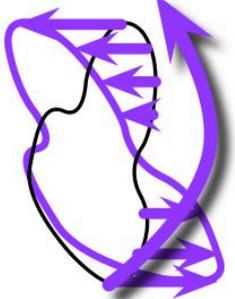
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stato tensionale per presso/tenso-flessione

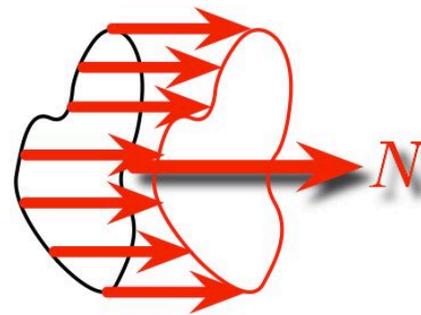
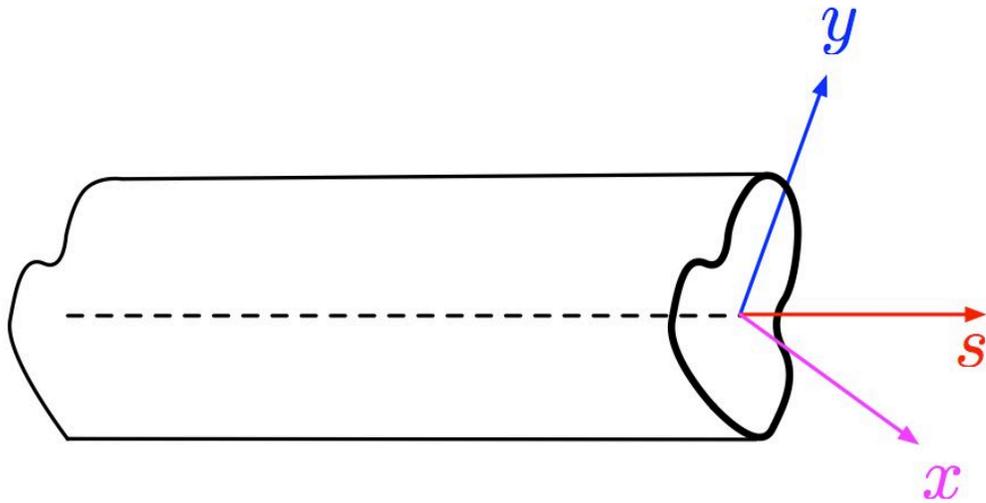


$$N = \int_A \sigma_n$$



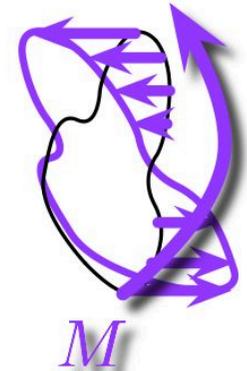
$$M = \int_A \sigma_m(y) y$$

# stato tensionale per presso/tenso-flessione



$$N = \int_A \sigma_n$$

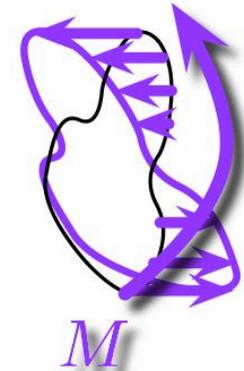
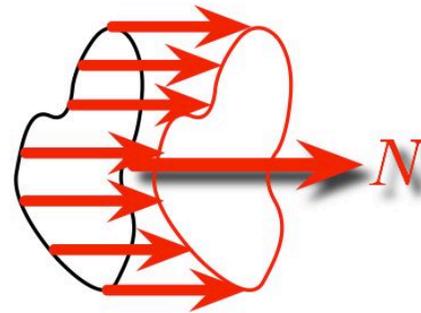
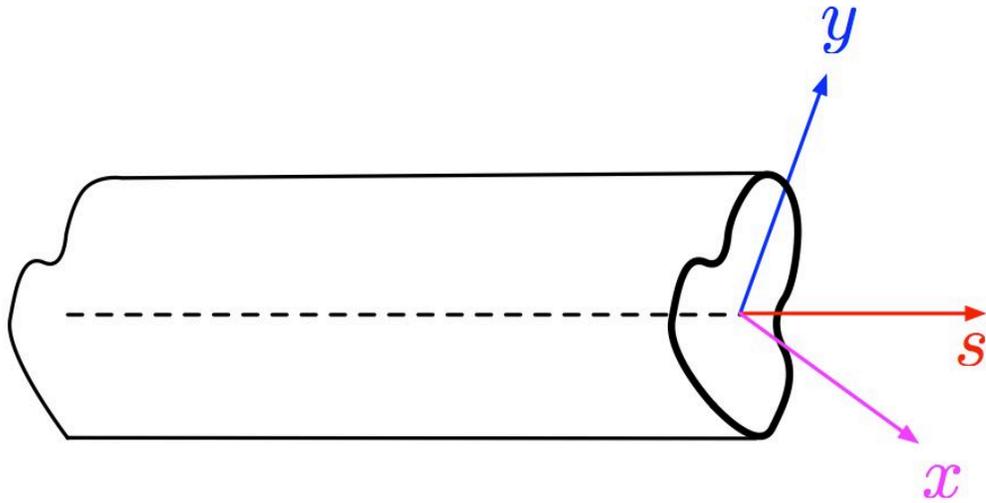
$$\int_A \sigma_m(y) = 0$$



$$M = \int_A \sigma_m(y) y$$

$$\int_A \sigma_n y = 0$$

# stato tensionale per presso/tenso-flessione



il sistema  $(N, M)$   
 è **EQUIVALENTE**  
 al sistema  $\sigma = \sigma_n - \sigma_m(y)$

$$N = \int_A \sigma_n$$

$$\int_A \sigma_m(y) = 0$$

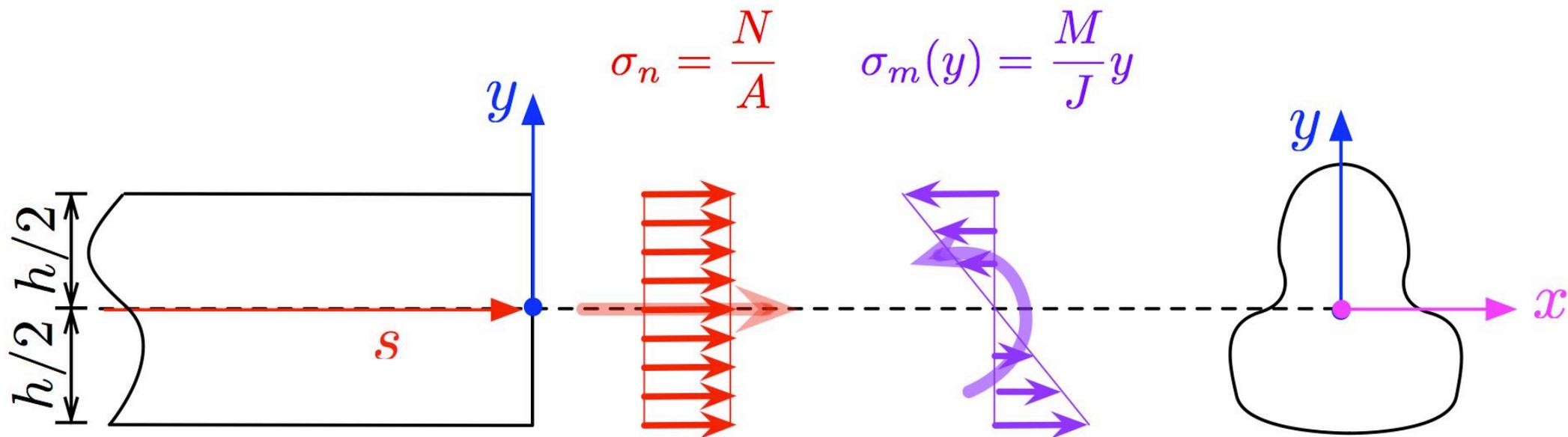
$$N = \int_A \sigma$$

$$M = \int_A \sigma_m(y) y$$

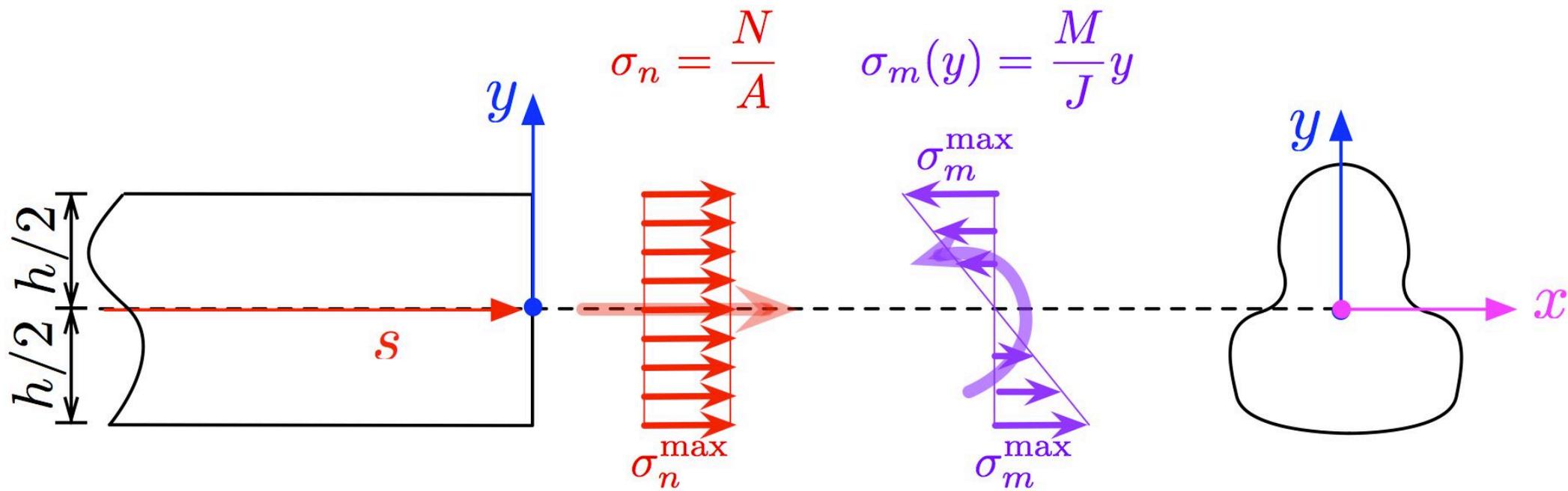
$$\int_A \sigma_n y = 0$$

$$M = \int_A \sigma y$$

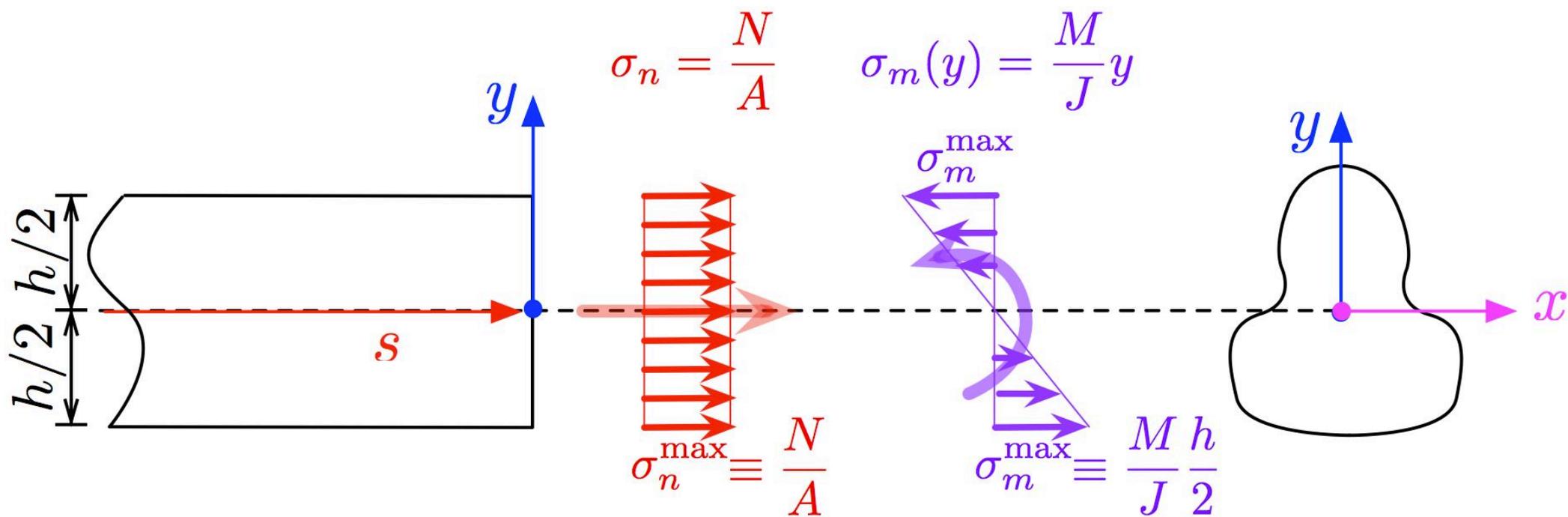
# progetto e verifica a presso/tenso-flessione



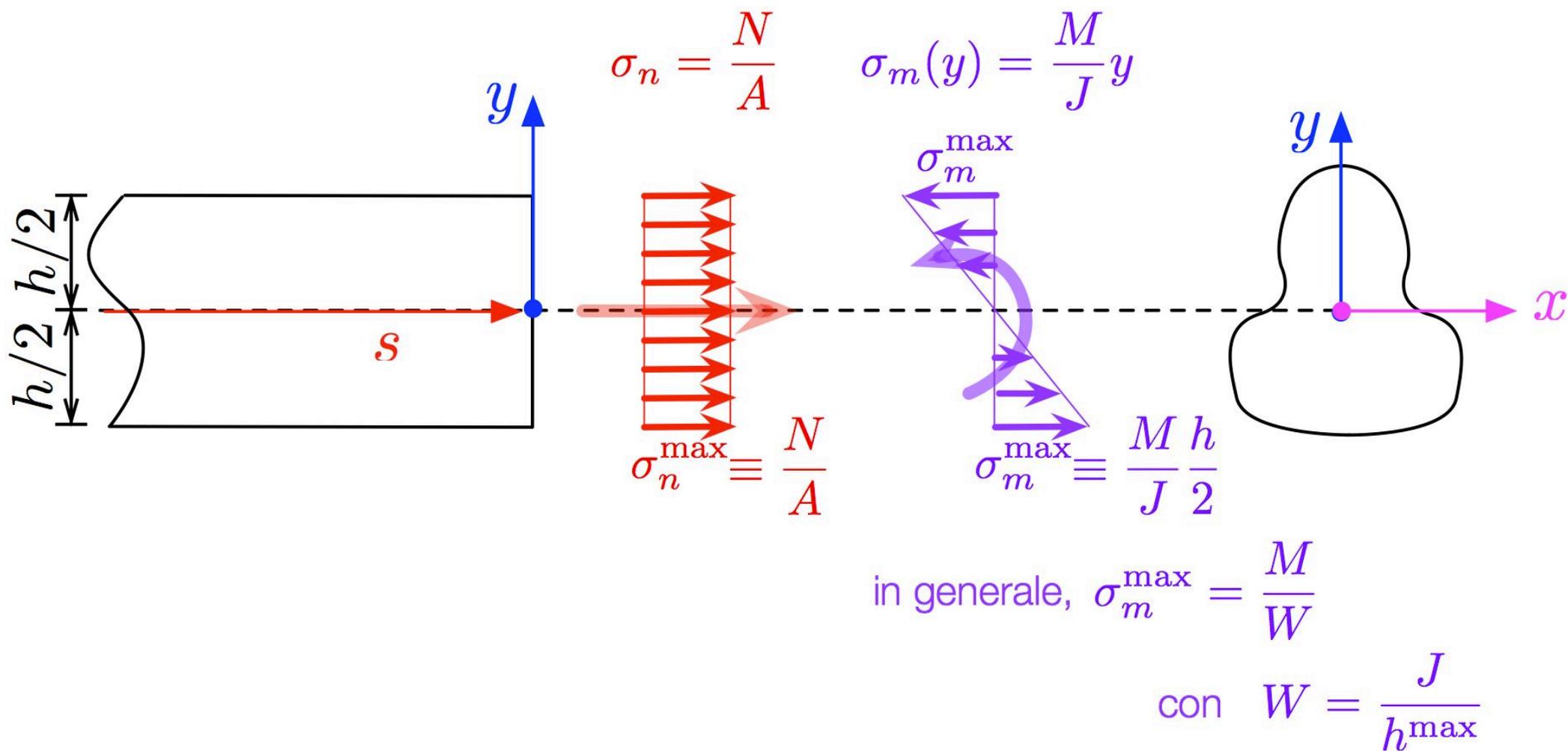
# progetto e verifica a presso/tenso-flessione



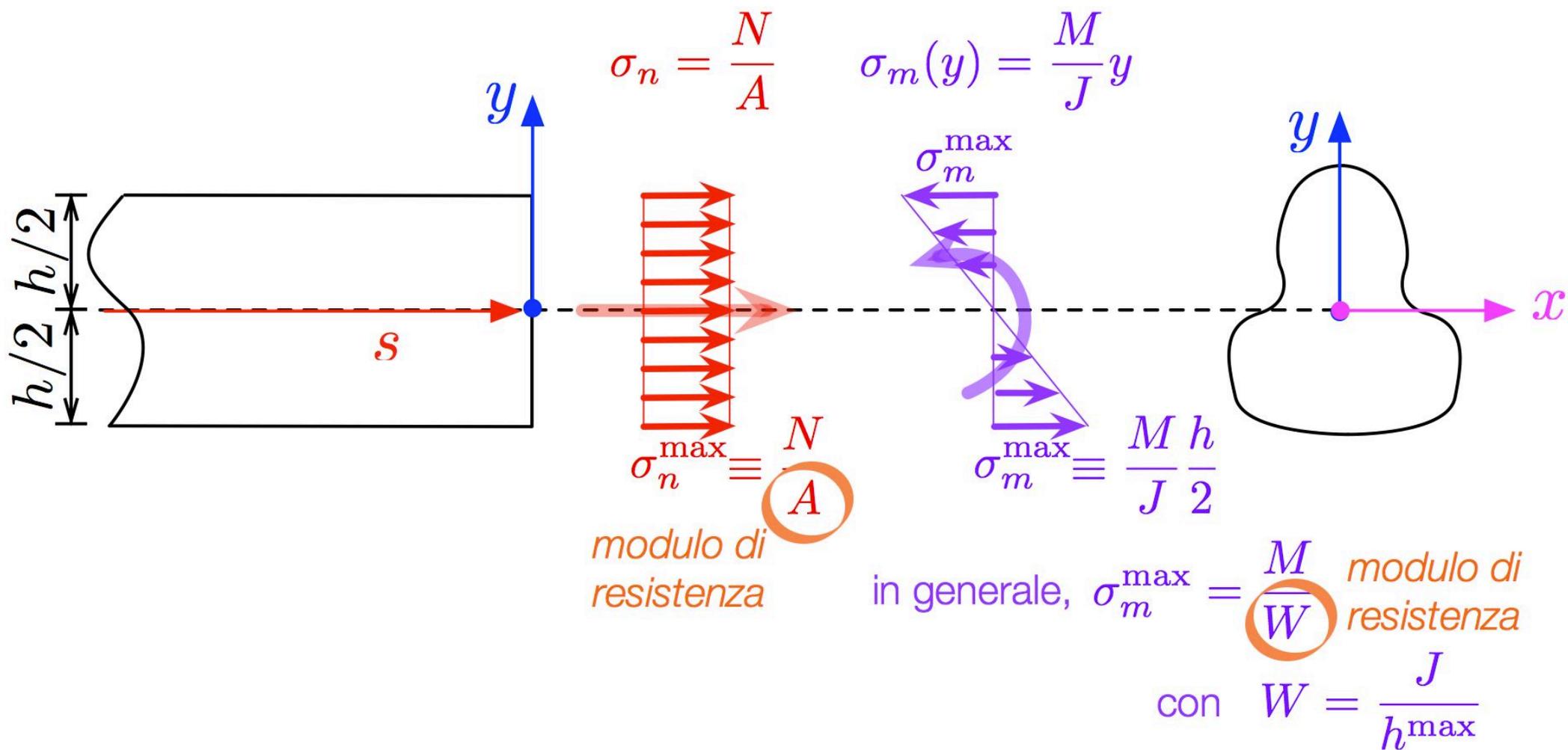
# progetto e verifica a presso/tenso-flessione



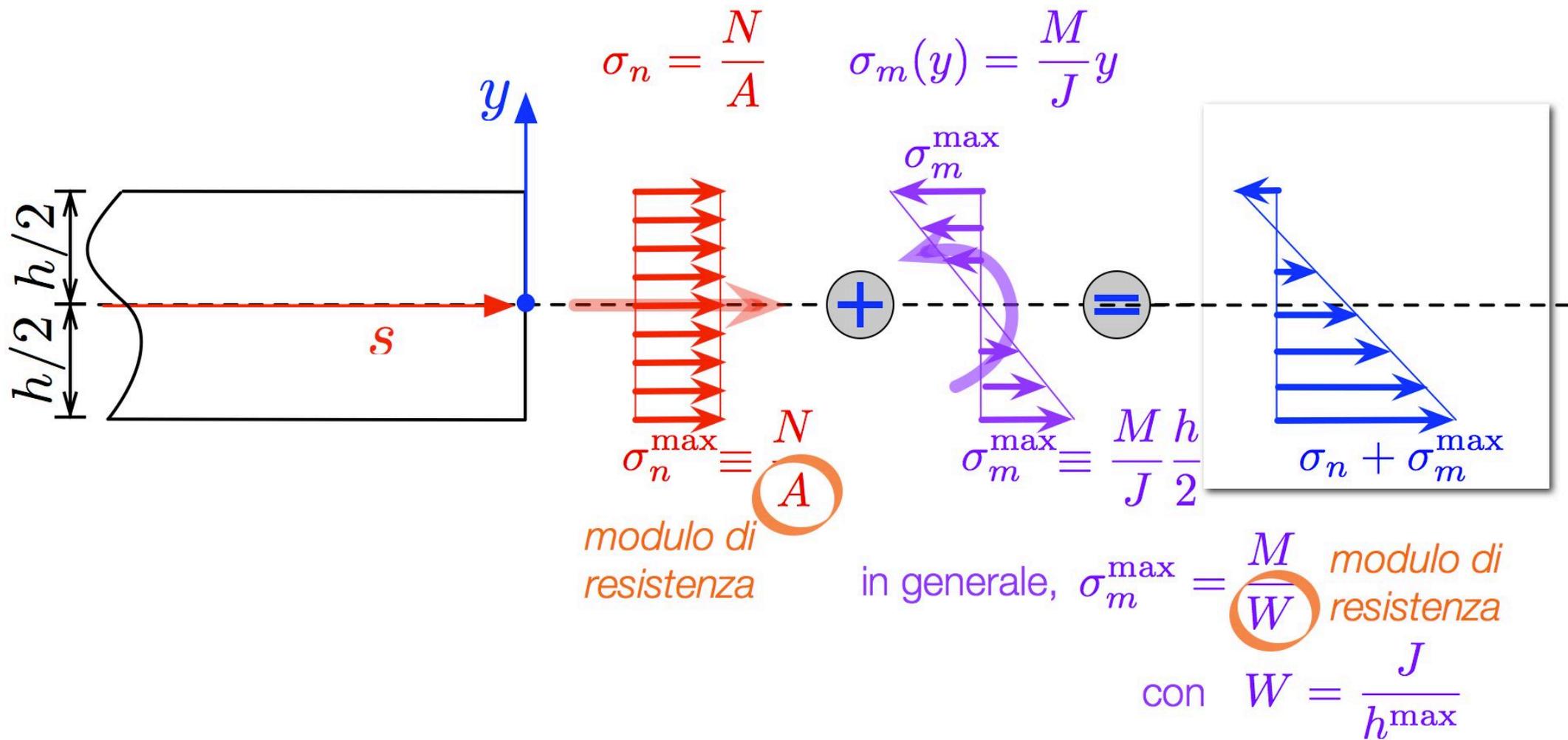
# progetto e verifica a presso/tenso-flessione



# progetto e verifica a presso/tenso-flessione

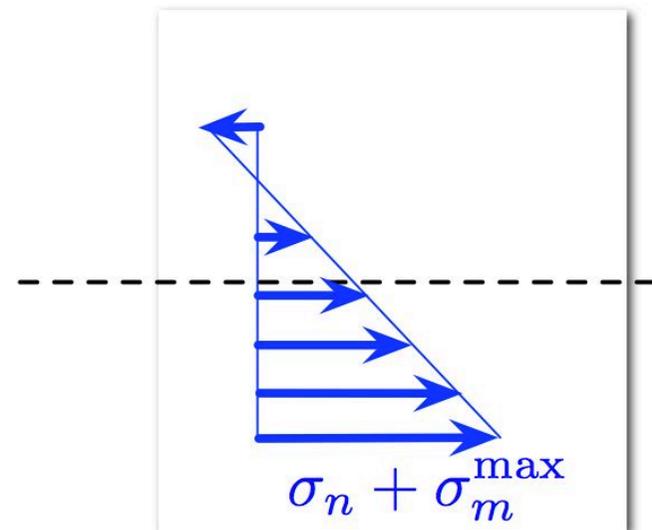


# progetto e verifica a presso/tenso-flessione



# progetto e verifica a presso/tenso-flessione

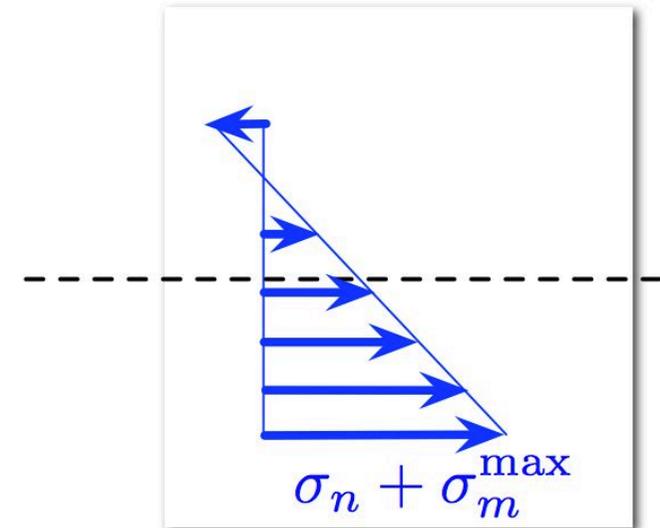
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$$\sigma_n = \frac{N}{A}$$
$$\sigma_m^{\max} = \frac{M}{W}$$

progetto e verifica a presso/tenso-flessione

per una **data sezione**,  
controllare lo stato tensionale massimo

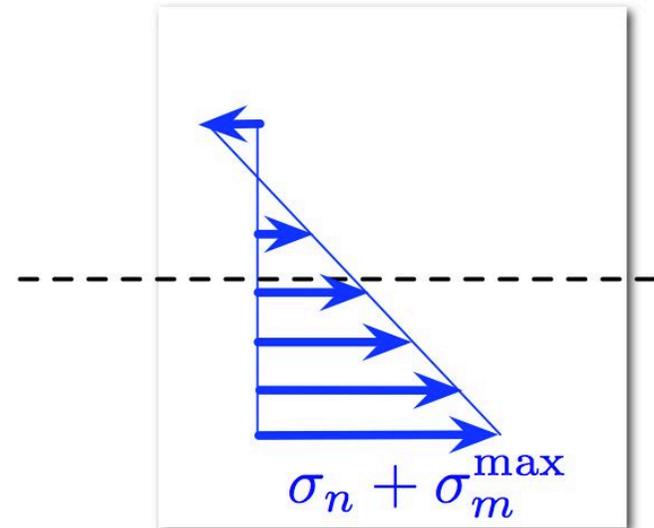


$$\sigma_n = \frac{N}{A}$$
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progetto e verifica a presso/tenso-flessione

per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \sigma_Y$$



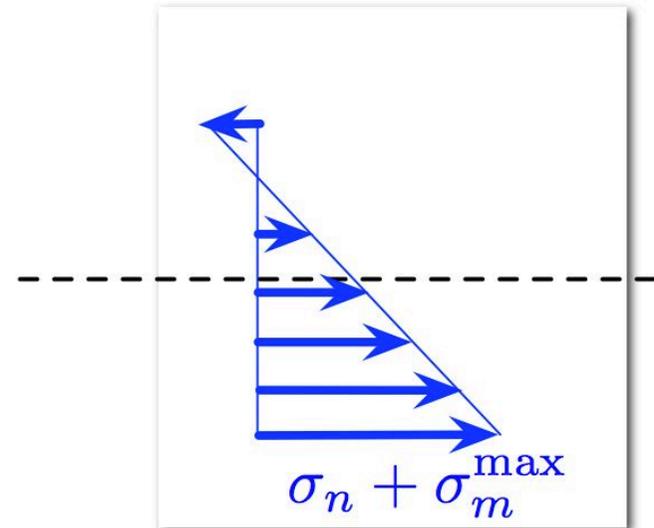
$$\sigma_n = \frac{N}{A}$$
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progetto e verifica a presso/tenso-flessione

per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \frac{\sigma_Y}{f_{cd}}$$

con un dato **fattore di sicurezza**  $f_{cd}$



$$\sigma_n = \frac{N}{A}$$
$$\sigma_m^{\max} = \frac{M}{W}$$

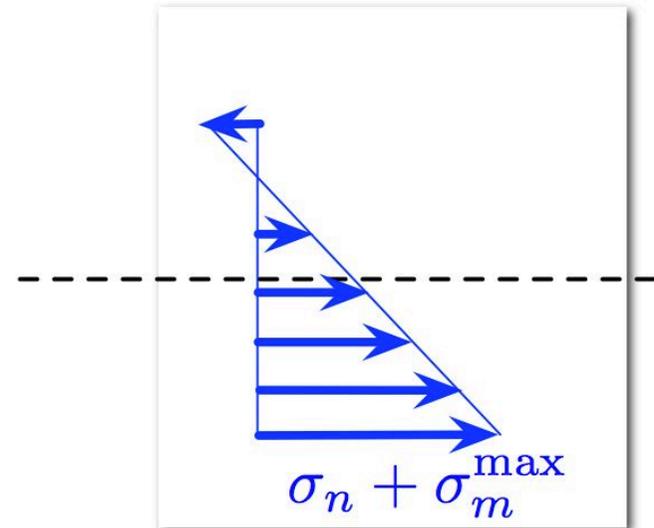
progetto e verifica a presso/tenso-flessione

per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \frac{\sigma_Y}{f_{cd}}$$

con un dato **fattore di sicurezza**  $f_{cd}$

trovare quella forma di sezione che  
**“massimizza”** lo stato tensionale (massimo)



$$\sigma_n = \frac{N}{A}$$
$$\sigma_m^{\max} = \frac{M}{W}$$

progetto e verifica a presso/tenso-flessione

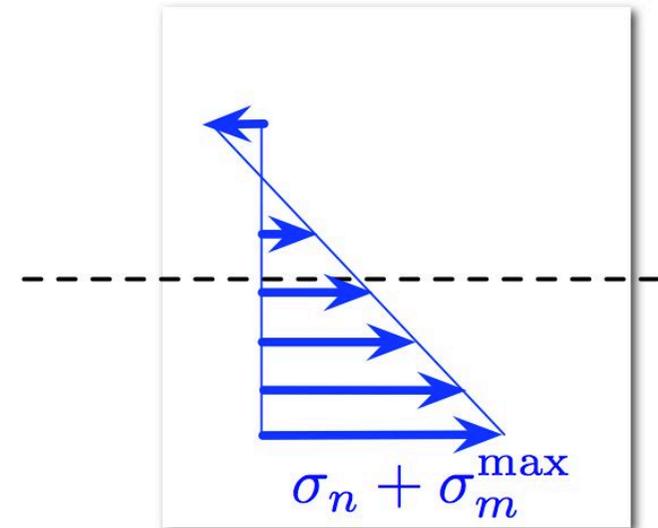
per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \frac{\sigma_Y}{f_{cd}}$$

con un dato **fattore di sicurezza**  $f_{cd}$

trovare quella forma di sezione che  
**“massimizzì”** lo stato tensionale (massimo)

$$\sigma_m^{\max} = \frac{M}{W}$$



$$\sigma_n = \frac{N}{A}$$

$$\sigma_m^{\max} = \frac{M}{W}$$

# progetto e verifica a presso/tenso-flessione

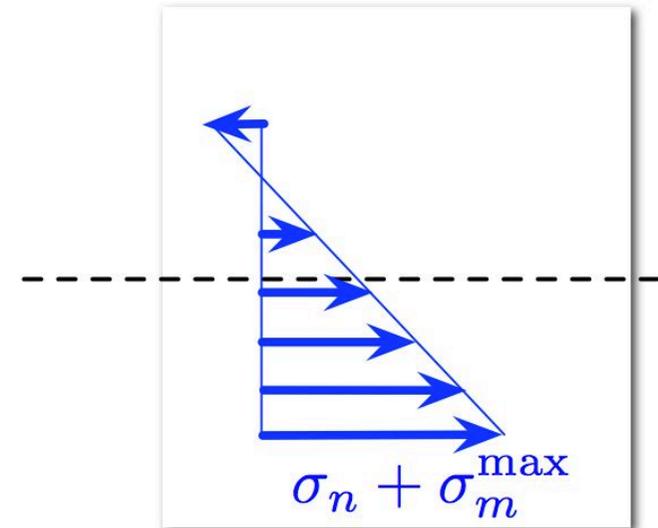
per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \frac{\sigma_Y}{f_{cd}}$$

con un dato **fattore di sicurezza**  $f_{cd}$

trovare quella forma di sezione che  
**“massimizzi”** lo stato tensionale (massimo)

$$\sigma_m^{\max} = \frac{M}{W} \equiv \sigma_Y$$



$$\sigma_n = \frac{N}{A}$$
$$\sigma_m^{\max} = \frac{M}{W}$$

progetto e verifica a presso/tenso-flessione

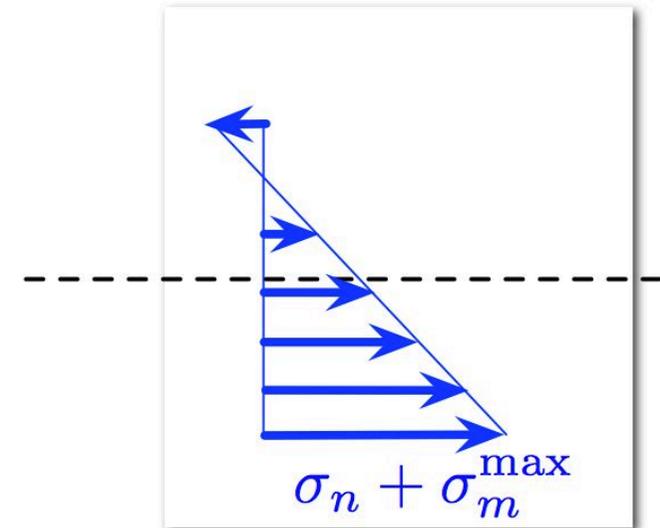
per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \frac{\sigma_Y}{f_{cd}}$$

con un dato **fattore di sicurezza**  $f_{cd}$

trovare quella forma di sezione che  
**“massimizzi”** lo stato tensionale (massimo)

$$\sigma_m^{\max} = \frac{M}{W} \equiv \sigma_Y \quad \Rightarrow \quad W^{\min} = \frac{M}{\sigma_Y}$$



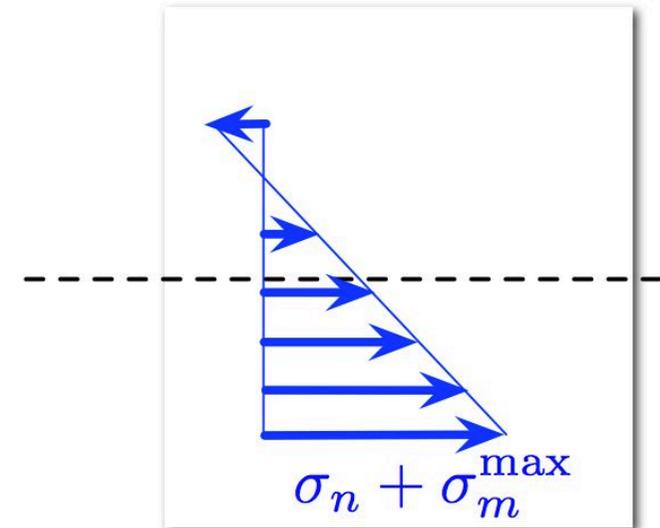
$$\sigma_n = \frac{N}{A}$$
$$\sigma_m^{\max} = \frac{M}{W}$$

# progetto e verifica a presso/tenso-flessione

per una **data sezione**,  
controllare lo stato tensionale massimo

$$\sigma_n + \sigma_m^{\max} \leq \frac{\sigma_Y}{\gamma}$$

con un dato **fattore di sicurezza**  $\gamma$



trovare quella forma di sezione che  
**“massimizzi”** lo stato tensionale (massimo)

$$\sigma_m^{\max} = \frac{M}{W} \equiv \sigma_Y$$



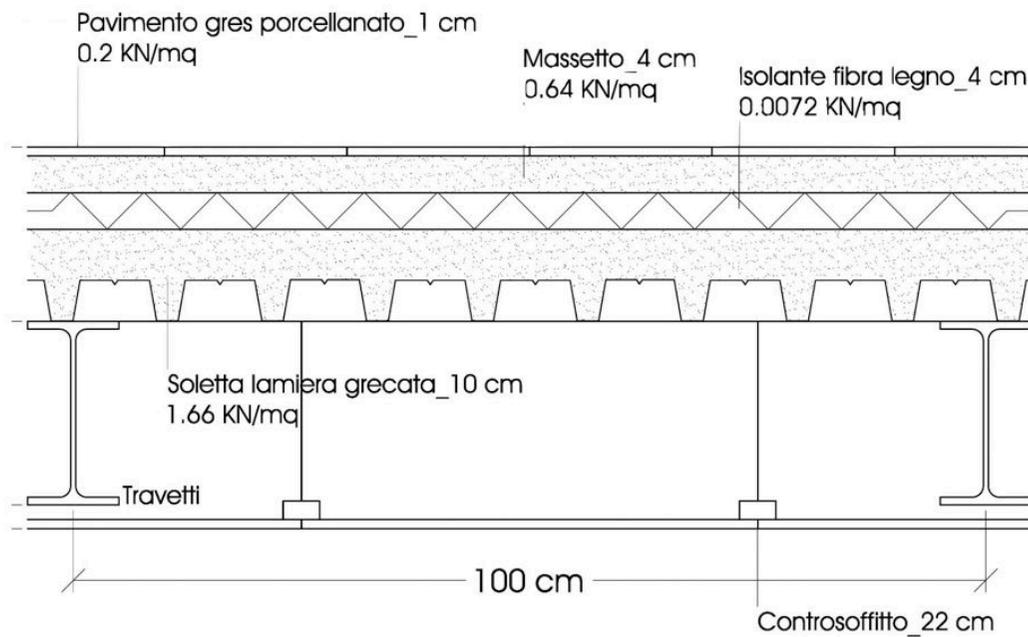
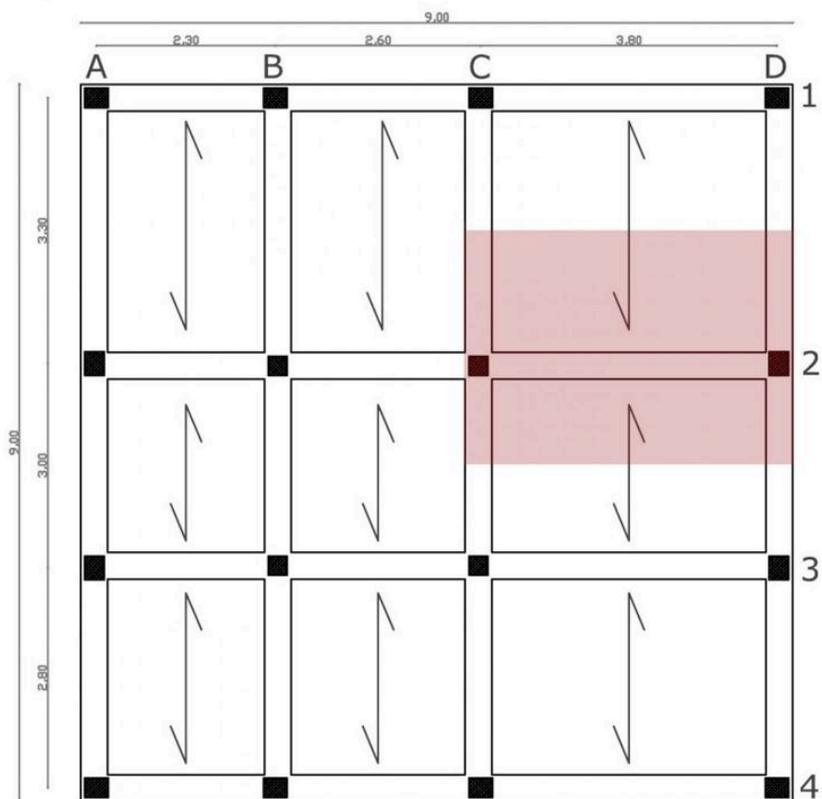
$$W^{\min} = \frac{M}{\sigma_Y}$$



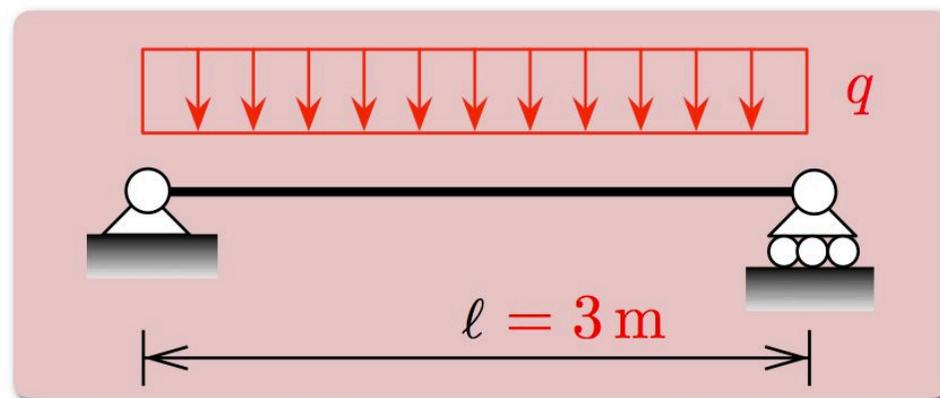
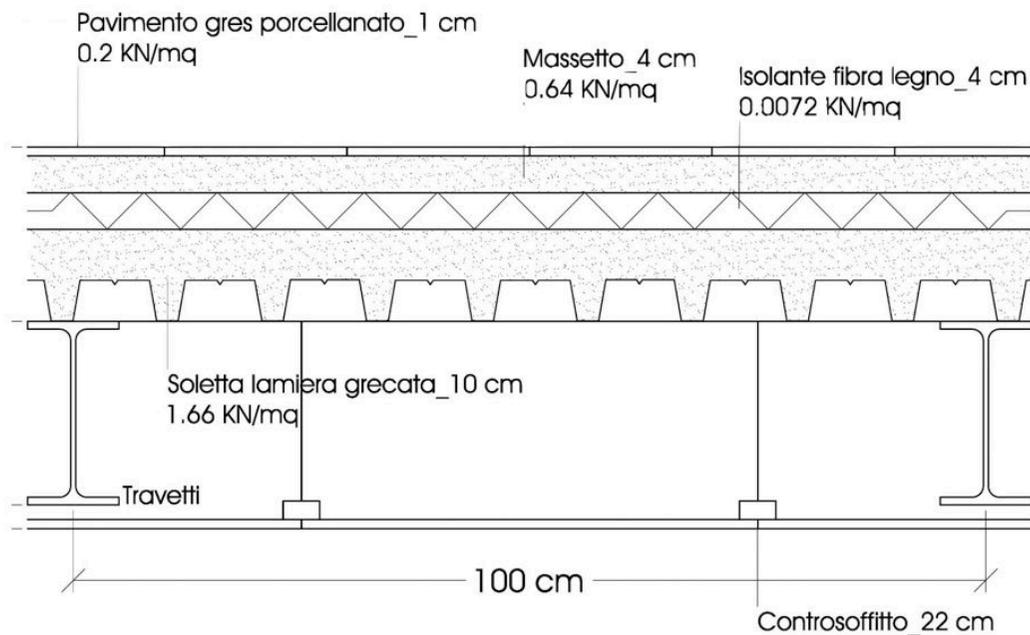
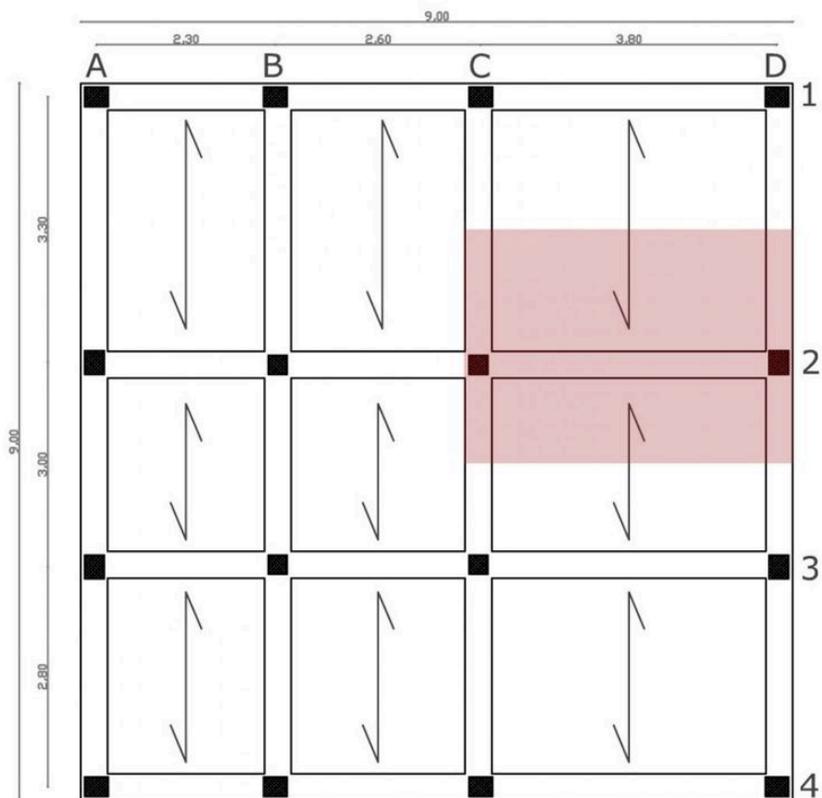
**progetto** di una sezione  
tale che abbia  $W \geq W^{\min}$

$$\sigma_n = \frac{N}{A}$$
$$\sigma_m^{\max} = \frac{M}{W}$$

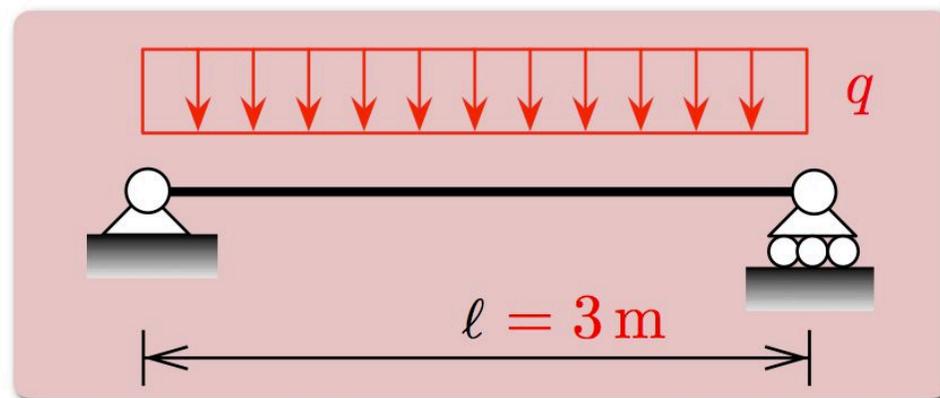
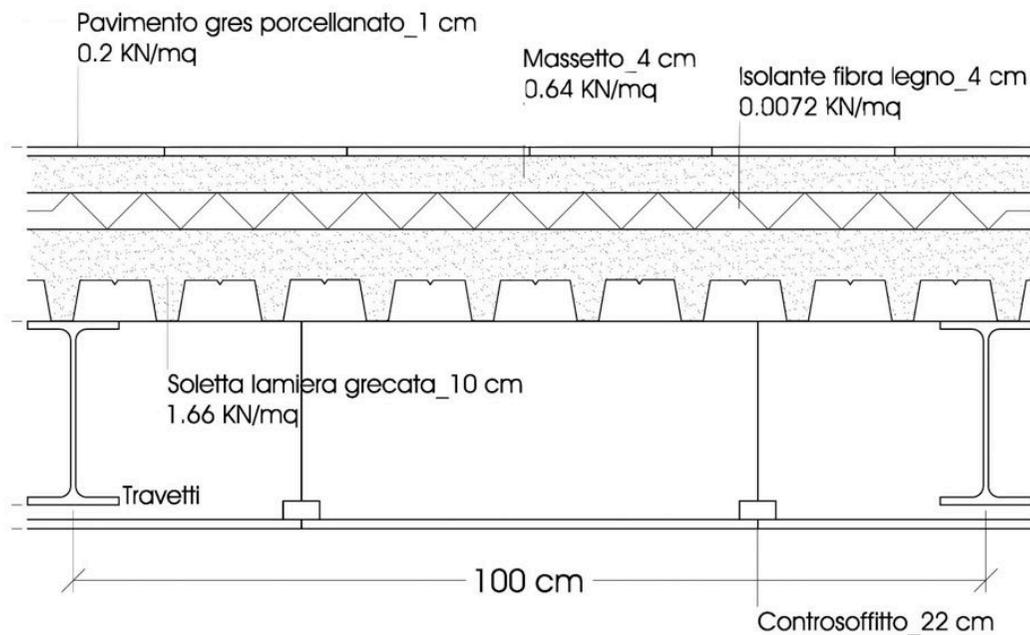
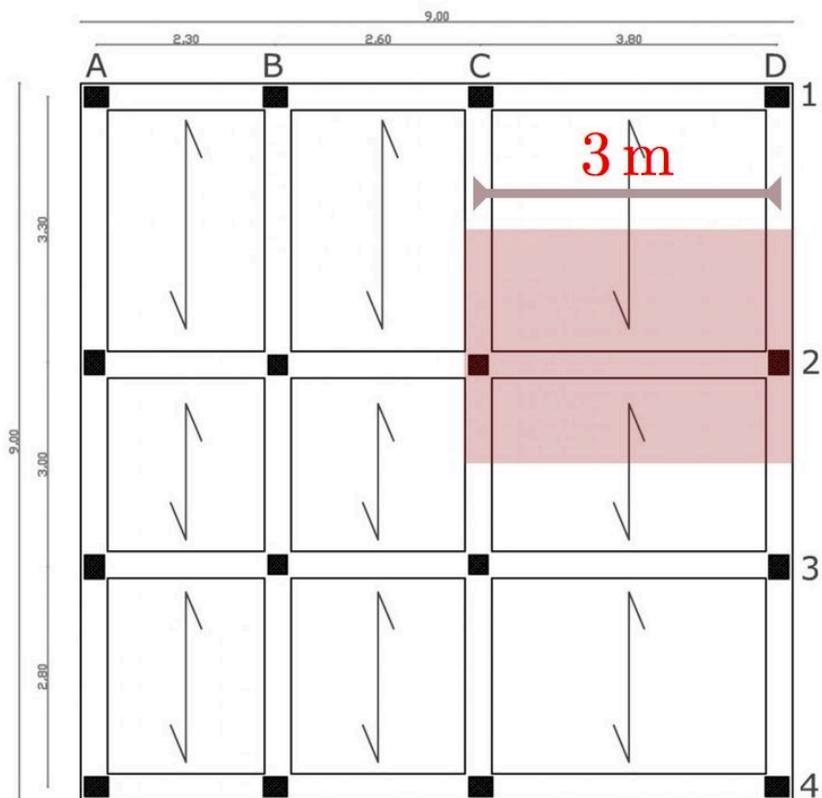
# progetto a flessione: esempio



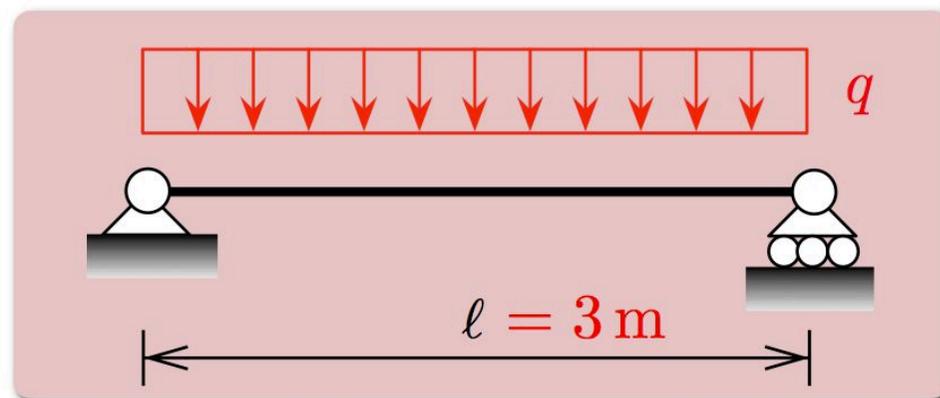
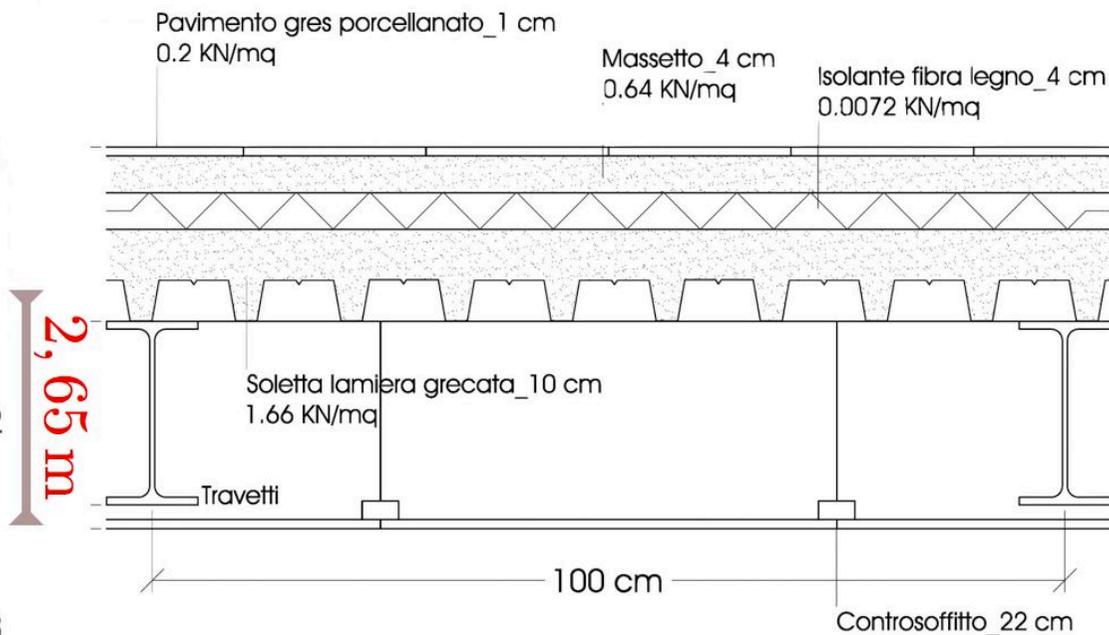
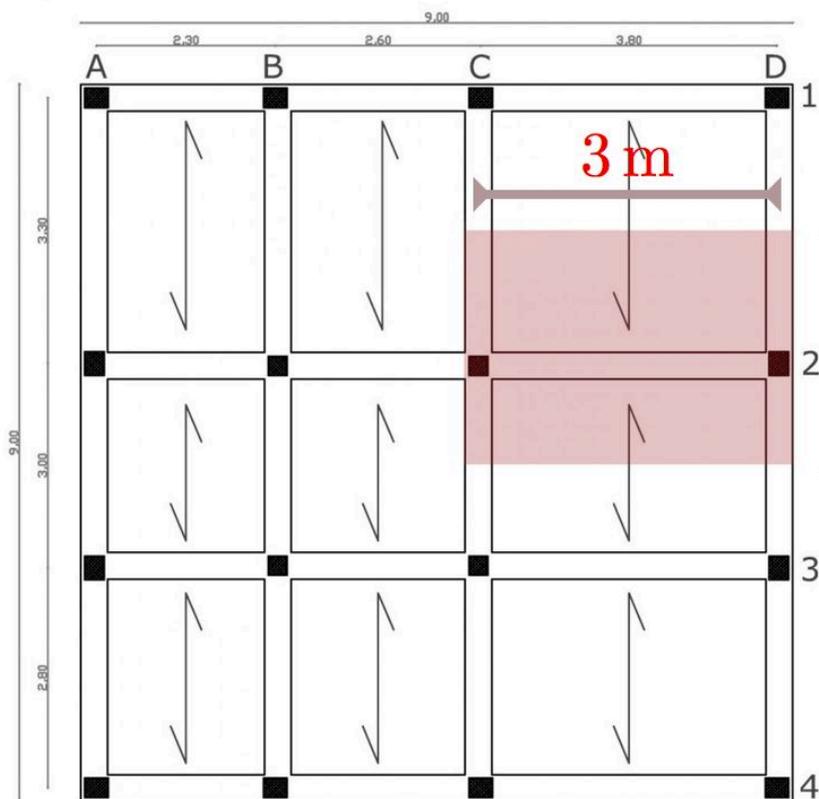
# progetto a flessione: esempio



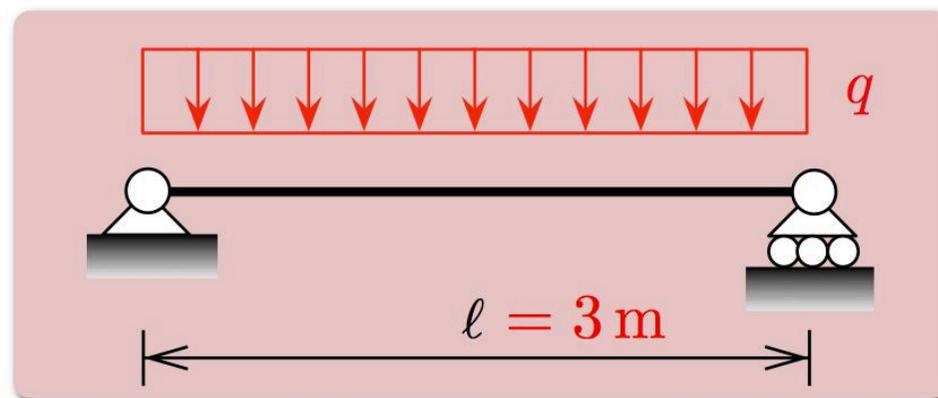
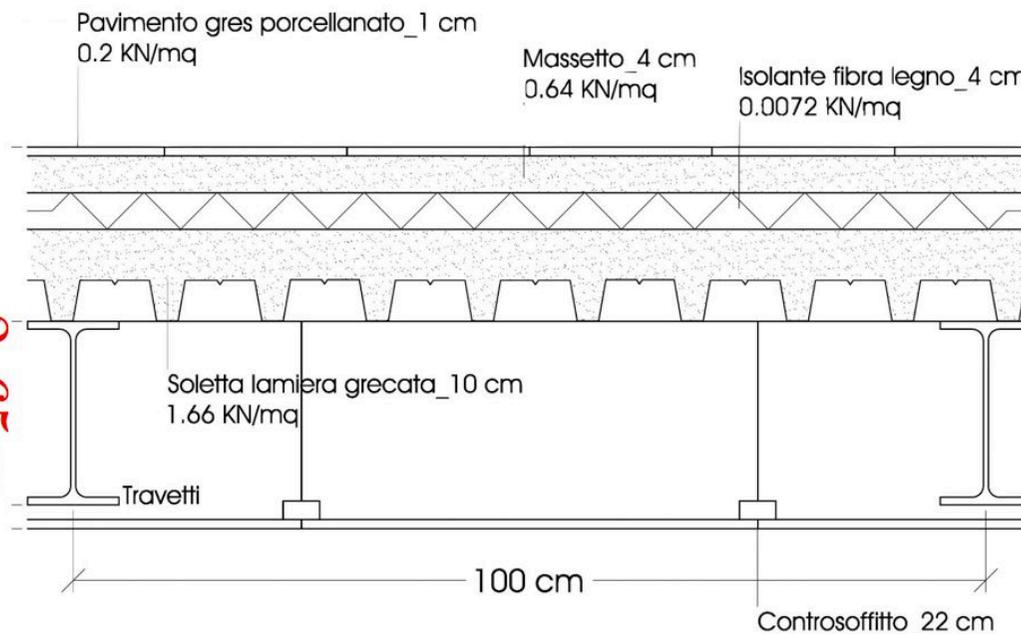
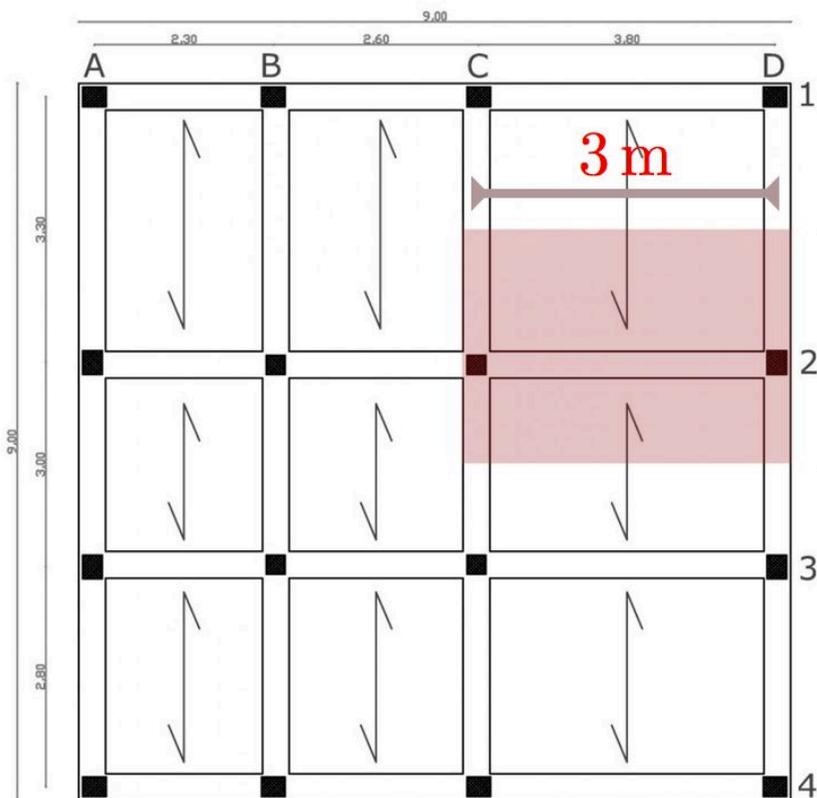
# progetto a flessione: esempio



# progetto a flessione: esempio

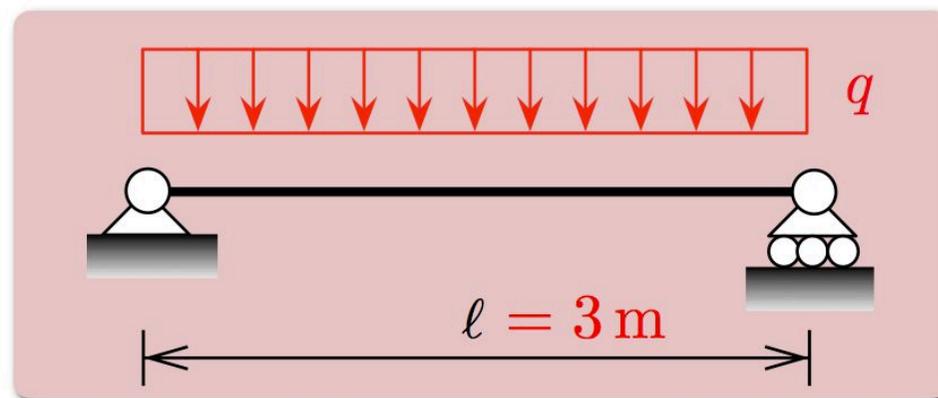
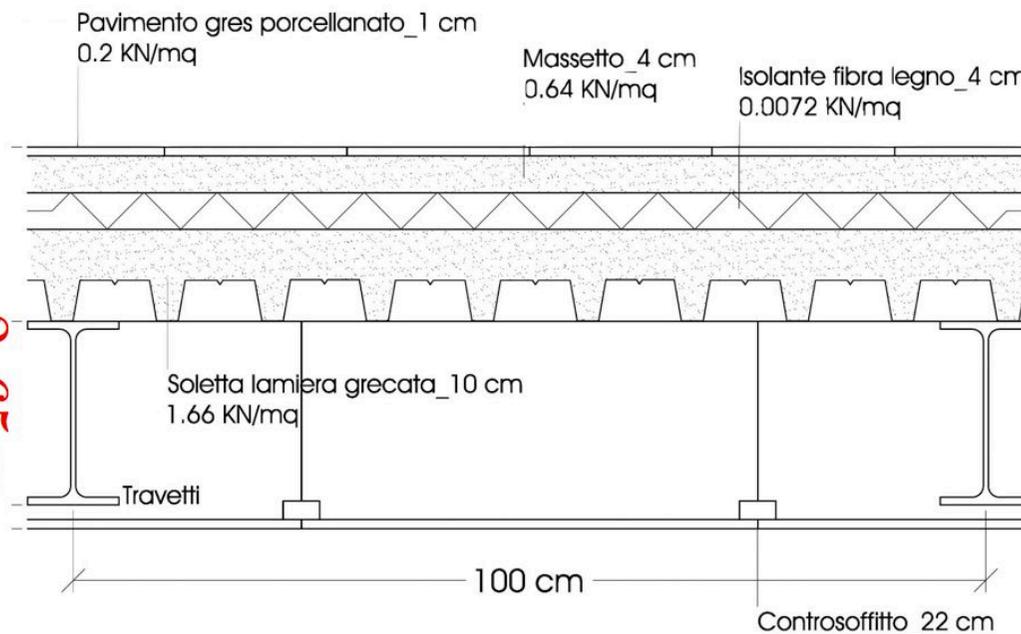
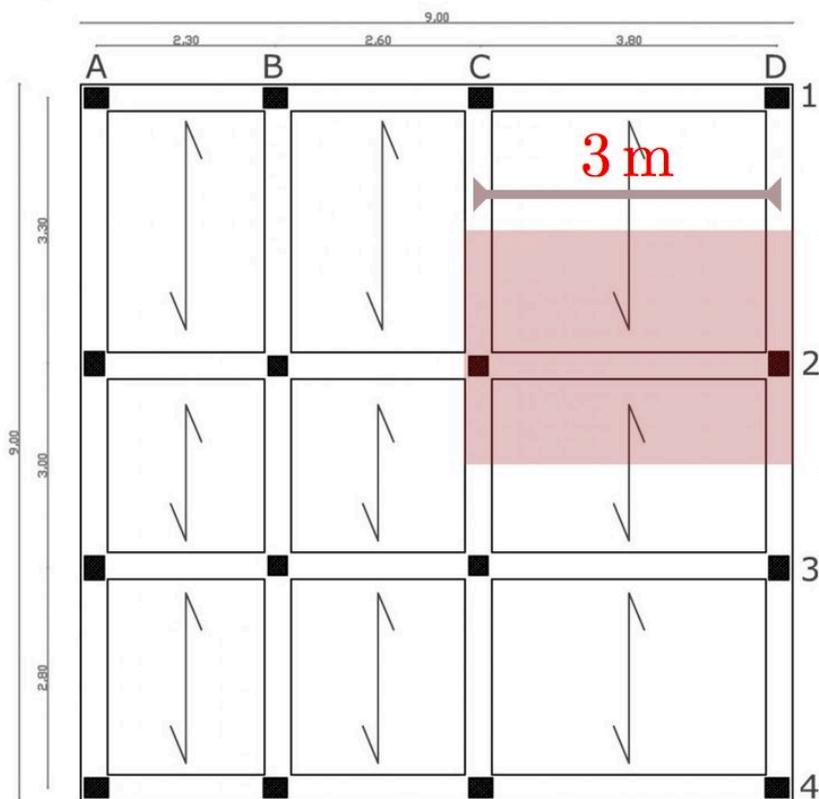


# progetto a flessione: esempio



carico stimato:  $10 \frac{\text{kN}}{\text{m}^2}$

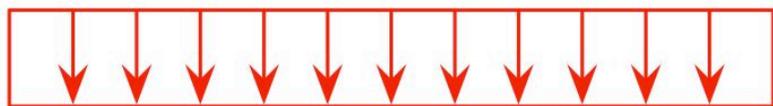
# progetto a flessione: esempio



carico stimato:  $10 \frac{\text{kN}}{\text{m}^2} \times 2,65\text{m} = q$

# progetto a flessione: esempio

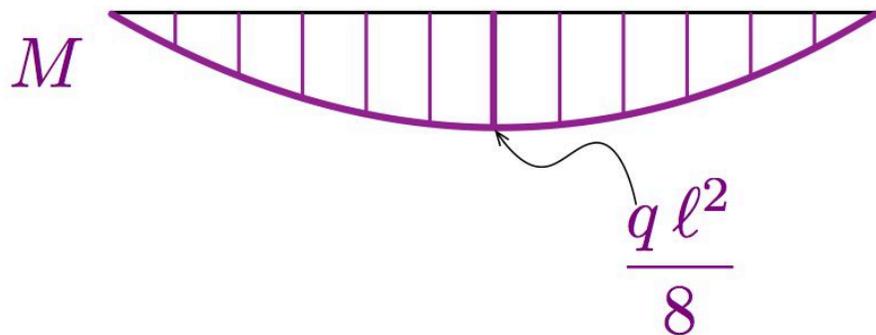
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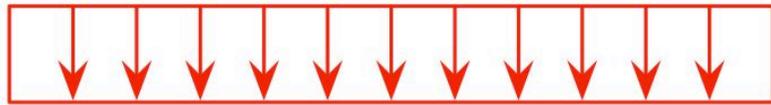
$$q = 10 \text{ kN/m}^2 \times 2,65 \text{ m} \approx 26,5 \text{ kN/m}$$



$$l = 3 \text{ m}$$



# progetto a flessione: esempio

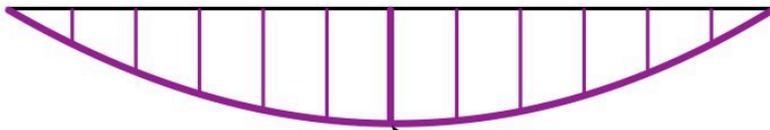


$$q = 10 \text{ kN/m}^2 \times 2,65 \text{ m} \approx 26,5 \text{ kN/m}$$



$$l = 3 \text{ m}$$

$M$

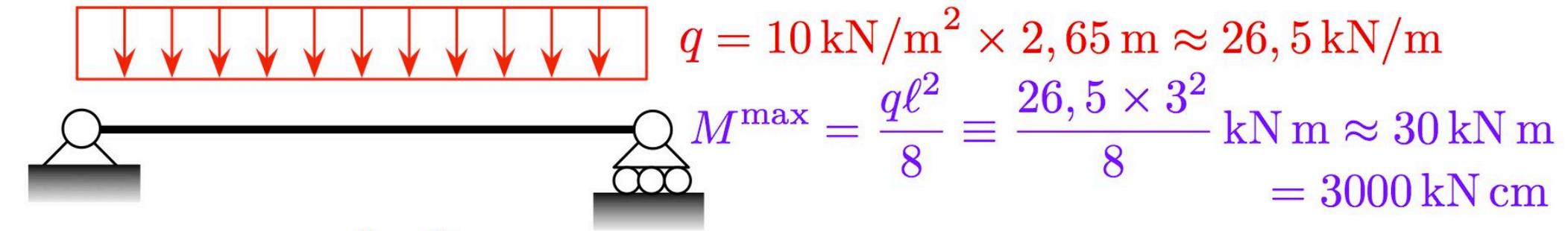


$$\frac{q l^2}{8}$$

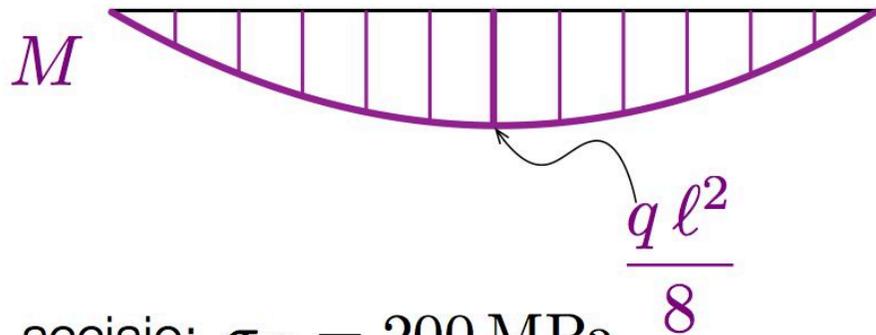
acciaio:  $\sigma_Y = 200 \text{ MPa}$

$$\text{progetto } W^{\min} = \frac{M^{\max}}{\sigma_Y}$$

## progetto a flessione: esempio

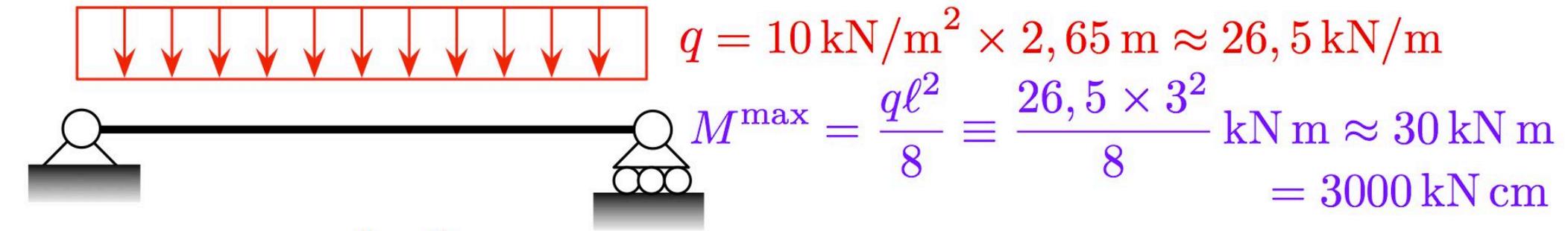


progetto  $W^{\min} = \frac{M^{\max}}{\sigma_Y}$



acciaio:  $\sigma_Y = 200 \text{ MPa}$   
 $\equiv 200 \cdot 10^6 \frac{\text{N}}{\text{m}^2} \equiv 200 \cdot 10^3 \frac{\text{kN}}{\text{m}^2} \equiv 20 \frac{\text{kN}}{\text{cm}^2}$

## progetto a flessione: esempio



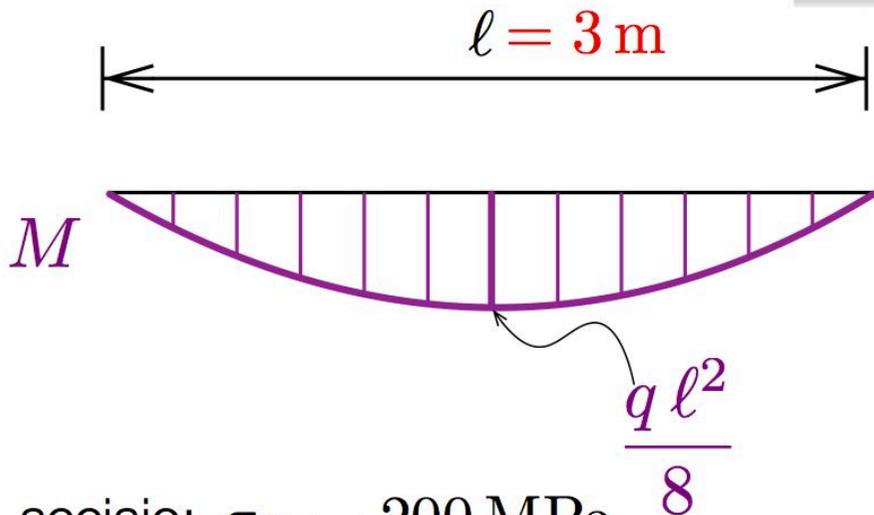
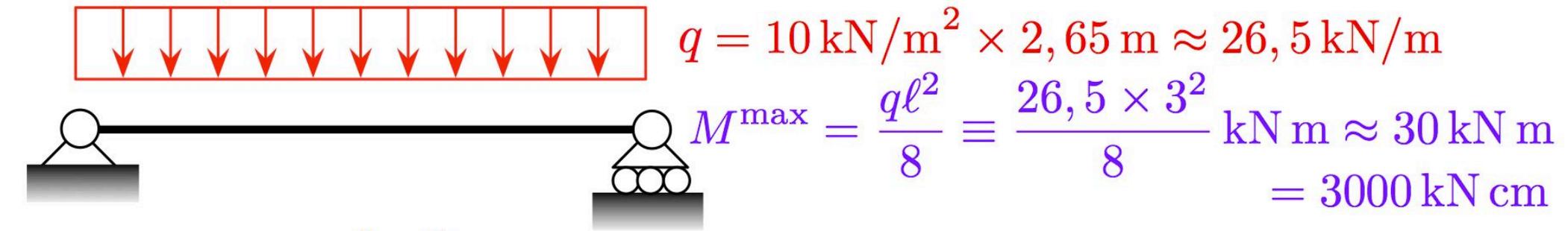
progetto  $W^{\min} = \frac{M^{\max}}{\sigma_Y}$

$$\equiv \frac{3000}{20} \text{ cm}^3 = 150 \text{ cm}^3$$

acciaio:  $\sigma_Y = 200 \text{ MPa}$

$$\equiv 200 \cdot 10^6 \frac{\text{N}}{\text{m}^2} \equiv 200 \cdot 10^3 \frac{\text{kN}}{\text{m}^2} \equiv 20 \frac{\text{kN}}{\text{cm}^2}$$

## progetto a flessione: esempio



progetto  $W^{\min} = \frac{M^{\max}}{\sigma_Y}$

$$\equiv \frac{3000}{20} \text{ cm}^3 = 150 \text{ cm}^3$$

acciaio:  $\sigma_Y = 200 \text{ MPa}$

$$\equiv 200 \cdot 10^6 \frac{\text{N}}{\text{m}^2} \equiv 200 \cdot 10^3 \frac{\text{kN}}{\text{m}^2} \equiv 20 \frac{\text{kN}}{\text{cm}^2}$$

➔ **IPE200**