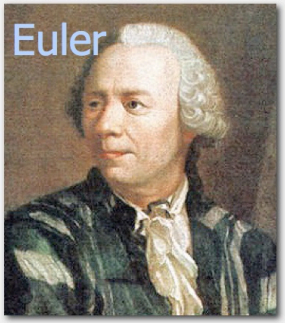
- problema elastico-lineare
  trave di Euler-Bernoulli
  tensioni assiali

- analisi a collasso (flessionale)
  analisi limite

- problema elastico-non lineare
  carico critico Euleriano



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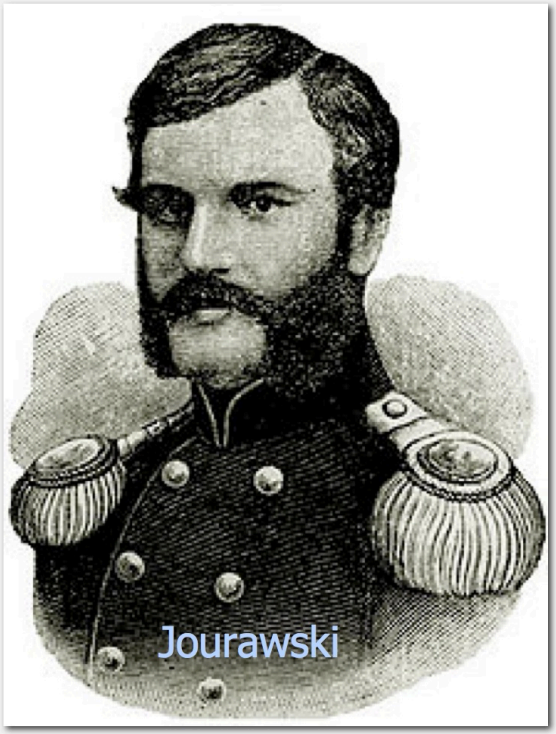

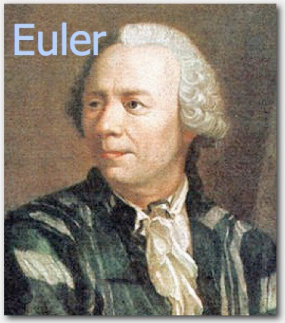

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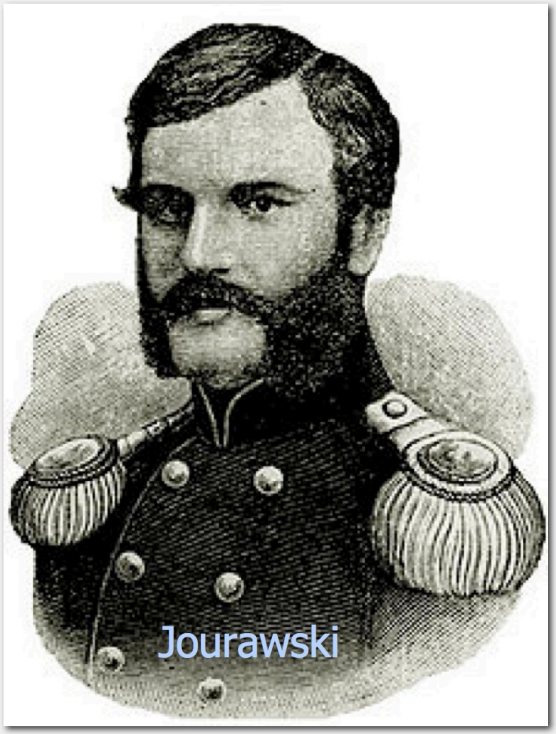
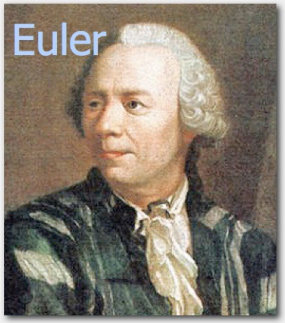
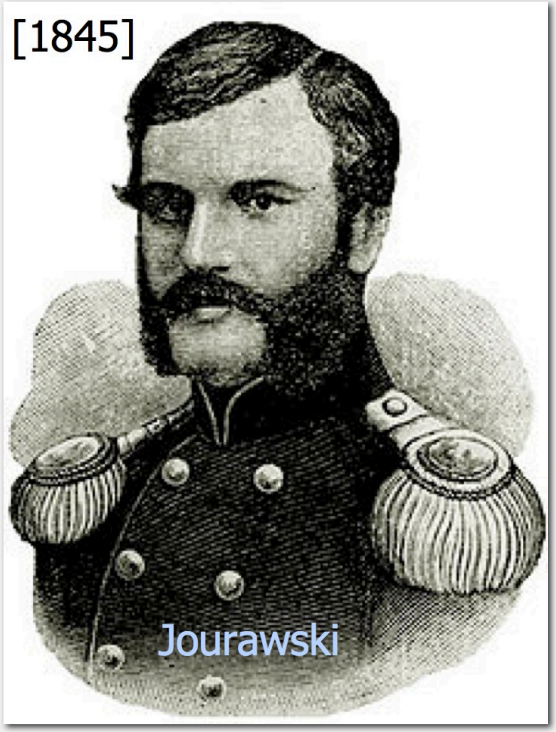
non linearità



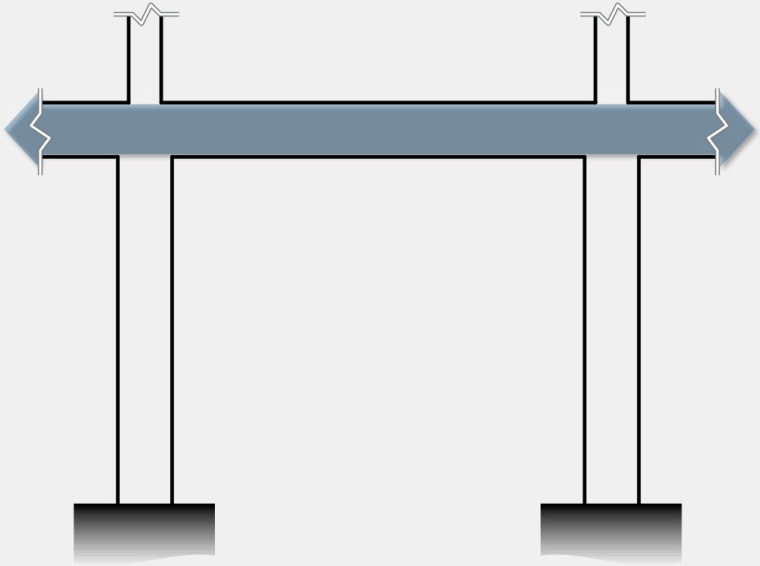



[1950]
Timoshenko

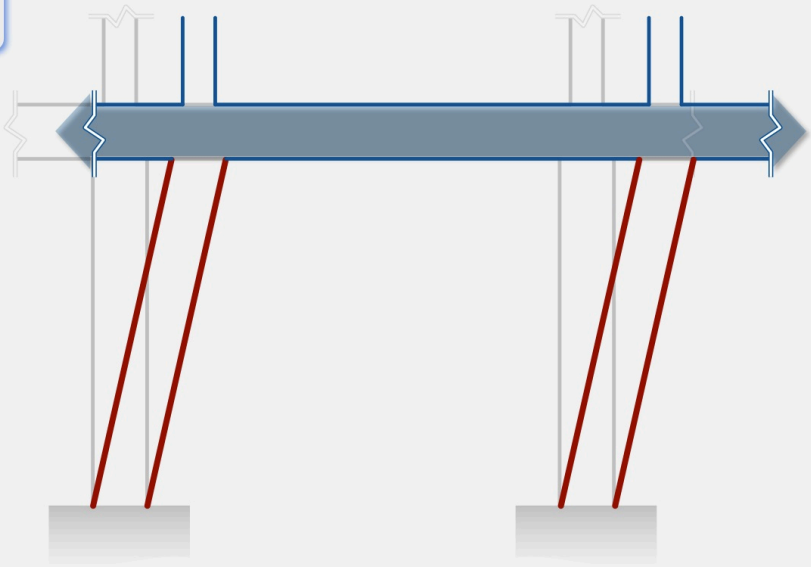

[1845]
Jourawski


Euler
[1750]


Bernoulli



il sisma "induce" un'energia di taglio (scorrimento)



Aquila 2009

Perù 2007

Perù 2007

Perù 2007

San Fernando (CA, USA) 1971

San Fernando (CA, USA) 1971

trave di Timoshenko

modello “esatto” del problema elastico-lineare

problema elastico-lineare della trave

equilibrio

$$\begin{cases} N'[s] + p[s] = 0 \\ T'[s] + q[s] = 0 \\ M'[s] + \mu[s] + T[s] = 0 \end{cases}$$

compatibilità

$$\begin{cases} \varepsilon[s] = u'[s] \\ \gamma[s] = v'[s] - \vartheta[s] = 0 \\ \chi[s] = \vartheta'[s] \end{cases}$$

legami costitutivi

$$\begin{cases} N[s] = EA\varepsilon[s] \\ M[s] = EJ\chi[s] \end{cases}$$

$$T[s] = \quad \gamma[s]$$

problema elastico-lineare della trave

equilibrio

$$\begin{cases} N'[s] + p[s] = 0 \\ T'[s] + q[s] = 0 \\ M'[s] + \mu[s] + T[s] = 0 \end{cases}$$

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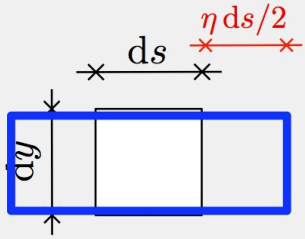
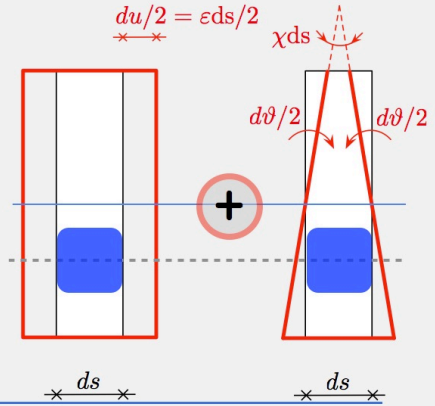
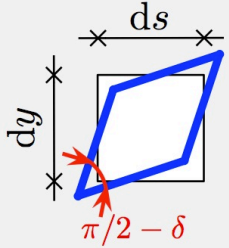
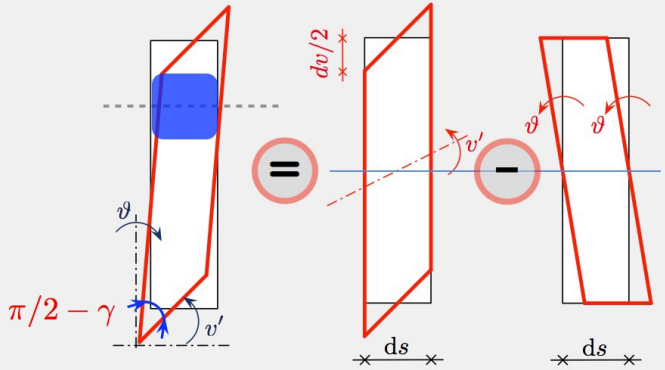
dimensioni $[\mathrm{F}] = \dfrac{[\mathrm{F}]}{[\mathrm{L}]^2}[\mathrm{L}]^2\,[1]$


ηds/2
ds
dy


du/2 = εds/2
χds
dϑ/2
dϑ/2
+
ds
ds

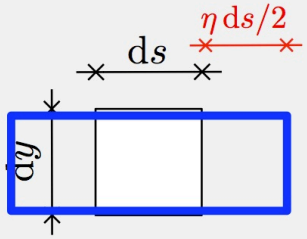

ds
dy
π/2 − δ


ϑ
π/2 − γ
v′
=
dv/2
v′
−
ϑ
ϑ
ds
ds

$$\int_A \sigma\dot{\eta}\,\mathrm{d}A = N\dot{\varepsilon} + M\dot{\chi}$$

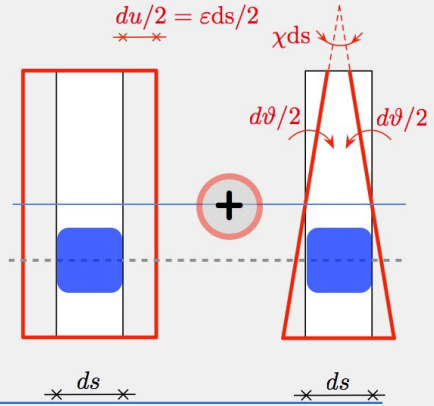
congruenza:

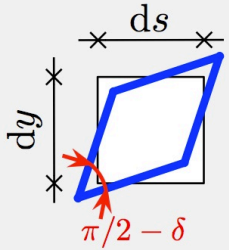
$$\eta(y) = \varepsilon \pm \chi\, y$$



test sperimentale:

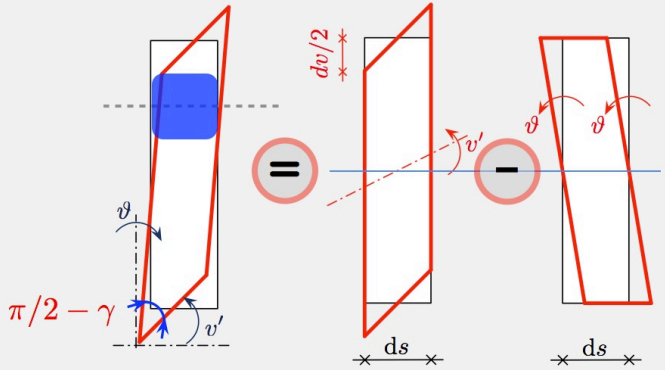
$$\sigma = E\,\eta$$





test sperimentale:

$$\tau = G\,\delta$$



$$\int_A \sigma\dot{\eta}\,\mathrm{d}A = N\dot{\varepsilon} + M\dot{\chi}$$

congruenza:

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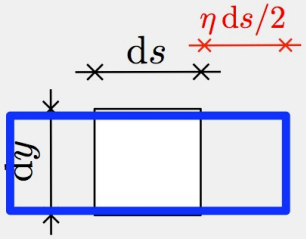
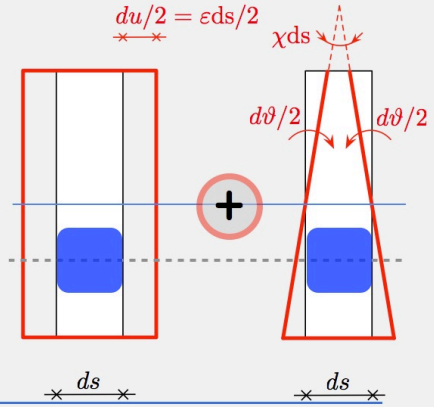
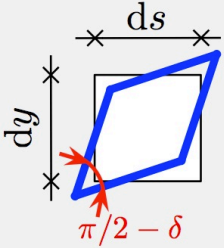


test sperimentale:

$$\sigma = E\,\eta$$

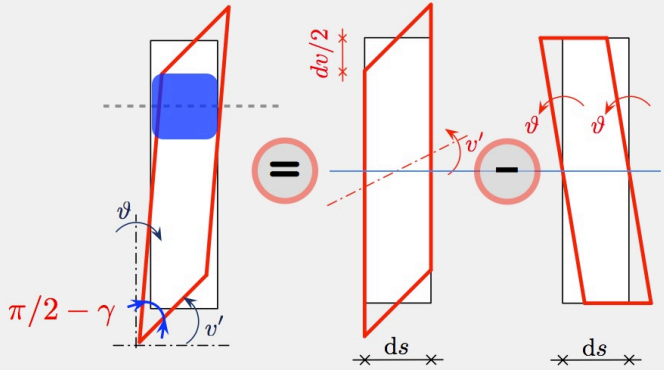


$$\delta(y) = \gamma$$



test sperimentale:

$$\tau = G\,\delta$$



equivalenza energetica

$$\int_A \sigma\,\varepsilon\,\mathrm{d}A = \int_A E\,\varepsilon^2\,\mathrm{d}A = EA\,\varepsilon^2$$

$$\int_A \frac{\sigma^2}{E}\,\mathrm{d}A = \frac{N^2}{EA}$$

ε e σ costanti su A (lungo y)

$$N = EA\,\varepsilon$$

equivalenza energetica

$$\int_A \sigma\,\varepsilon\,\mathrm{d}A = \begin{cases} \displaystyle\int_A E\,\varepsilon^2\,\mathrm{d}A = EA\,\varepsilon^2 \\ \displaystyle\int_A \frac{\sigma^2}{E}\,\mathrm{d}A = \frac{N^2}{EA} \end{cases}$$

ε e σ costanti su A (lungo y)

$$N = EA\,\varepsilon$$

$$\int_A \tau\,\delta\,\mathrm{d}A = \begin{cases} \displaystyle\int_A G\delta^2\,\mathrm{d}A =: GA^\star\,\gamma^2 \\ \displaystyle\int_A \frac{\tau^2}{G}\,\mathrm{d}A =: \frac{T^2}{GA^\star} \end{cases}$$

$$T = GA^\star\,\gamma$$

area "ridotta" a taglio $A^\star$

---

sezioni rettangolari $A^\star = 5/6\, A$

$$\int_A \tau\, \delta \, \mathrm{d}A = \begin{cases} \displaystyle\int_A G\delta^2 \, \mathrm{d}A =: GA^\star \, \gamma^2 \\ \displaystyle\int_A \frac{\tau^2}{G} \, \mathrm{d}A \; =: \frac{T^2}{GA^\star} \end{cases}$$

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τ nota per equilibrio (Jourawski)

$$A^\star = \frac{T^2}{\displaystyle\int_A \tau^2 \, \mathrm{d}A}$$

area "ridotta" a taglio $A^\star$

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$$T = GA^\star \gamma$$

disuguaglianza di Cauchy-Schwarz

$$\big(\text{media}(\tau)\big)^2 \leq \text{media}(\tau^2)$$

τ nota per equilibrio (Jourawski)

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area "ridotta" a taglio $A^\star$

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disuguaglianza di Cauchy-Schwarz

$$\big(\text{media}(\tau)\big)^2 \leq \text{media}(\tau^2)$$

$$\text{media}(\tau) = \frac{1}{A}\int_A \tau \,\mathrm{d}A \equiv \frac{T}{A}$$

$$\text{media}(\tau^2) = \frac{1}{A}\int_A \tau^2 \,\mathrm{d}A \equiv \frac{1}{A}\frac{T^2}{A^\star}$$

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$$T = GA^\star\,\gamma$$

disuguaglianza di Cauchy-Schwarz

$$\big(\text{media}(\tau)\big)^2 \leq \text{media}(\tau^2)$$

$A^\star \leq A$

τ nota per equilibrio (Jourawski)

$$\text{media}(\tau) = \frac{1}{A}\int_A \tau \,\mathrm{d}A \equiv \frac{T}{A}$$

$$\text{media}(\tau^2) = \frac{1}{A}\int_A \tau^2 \,\mathrm{d}A \equiv \frac{1}{A}\frac{T^2}{A^\star}$$

$$A^\star = \frac{T^2}{\displaystyle\int_A \tau^2 \,\mathrm{d}A}$$

area "ridotta" a taglio $A^\star$

---

sezioni rettangolari $A^\star = 5/6\, A$

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$$T = GA^\star\,\gamma$$

confronto tra modelli di trave

Euler-Bernoulli
vs
Timoshenko

equilibrio

$$\begin{cases} N'[s] + p[s] = 0 \\ T'[s] + q[s] = 0 \\ M'[s] + \mu[s] + T[s] = 0 \end{cases}$$

compatibilità

$$\begin{cases} \varepsilon[s] = u'[s] \\ \gamma[s] = v'[s] - \vartheta[s] \\ \chi[s] = \vartheta'[s] \end{cases}$$

legami costitutivi

$$\begin{cases} N[s] = EA\,\varepsilon[s] \\ T[s] = GA^{\star}\gamma[s] \\ M[s] = EJ\,\chi[s] \end{cases}$$

equilibrio

$$\begin{cases} N'[s] + p[s] = 0 \\ T'[s] + q[s] = 0 \\ M'[s] + \mu[s] + T[s] = 0 \end{cases}$$

compatibilità

$$\begin{cases} \varepsilon[s] = u'[s] \\ \gamma[s] = v'[s] - \vartheta[s] \\ \chi[s] = \vartheta'[s] \end{cases} \quad \gamma[s] = 0$$

legami costitutivi

$$\begin{cases} N[s] = EA\,\varepsilon[s] \\ T[s] = GA^\star \gamma[s] \\ M[s] = EJ\,\chi[s] \end{cases}$$

$GA^\star \to \infty$

il modello di Timoshenko "contiene" quello di Euler-Bernoulli

E.-B. introduce un errore in T. perché trascura energia

$$\frac{T^2}{GA^\star} << \frac{M^2}{EJ}$$

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$$\frac{T^2}{GA^\star} << \frac{M^2}{EJ}$$

per ordini di grandezza

$$M \approx T\,\ell$$
$$G \approx E$$
$$A^\star \approx A$$

E.-B. introduce un errore in T. perché trascura energia

$$\frac{T^2}{GA^\star} << \frac{M^2}{EJ}$$

per ordini di grandezza

$$M \approx T\,\ell$$
$$G \approx E$$
$$A^\star \approx A$$

$$\frac{T^2}{EA} << \frac{T^2\,\ell^2}{EJ}$$

E.-B. introduce un errore in T. perché trascura energia

$$\frac{T^2}{GA^\star} << \frac{M^2}{EJ}$$

per ordini di grandezza

$$M \approx T\,\ell$$
$$G \approx E$$
$$A^\star \approx A$$

$$\frac{T^2}{EA} << \frac{T^2\,\ell^2}{EJ}$$

$$\frac{\ell^2}{J/A} >> 1$$

errore basso per travi snelle

E.-B. introduce un errore in T. perché trascura energia

$$\frac{T^2}{GA^\star} << \frac{M^2}{EJ}$$

per ordini di grandezza

$$M \approx T\,\ell \qquad G \approx E \qquad A^\star \approx A$$

$$\frac{\cancel{T^2}}{\cancel{E}A} << \frac{\cancel{T^2}\,\ell^2}{\cancel{E}J}$$

$$\frac{\ell^2}{J/A} >> 1$$

errore basso per travi snelle

come per confronto travi iso/iper-statica

$$\int_\ell \frac{N_{\mathrm{eb}}^2}{EA} + \frac{M_{\mathrm{eb}}^2}{EJ} \geq \int_\ell \frac{N_{\mathrm{t}}^2}{EA} + \frac{T_{\mathrm{t}}^2}{GA^\star} + \frac{M_{\mathrm{t}}^2}{EJ}$$

a parità di equilibrio, errore a vantaggio di sicurezza (tensioni sovrastimate in media)