

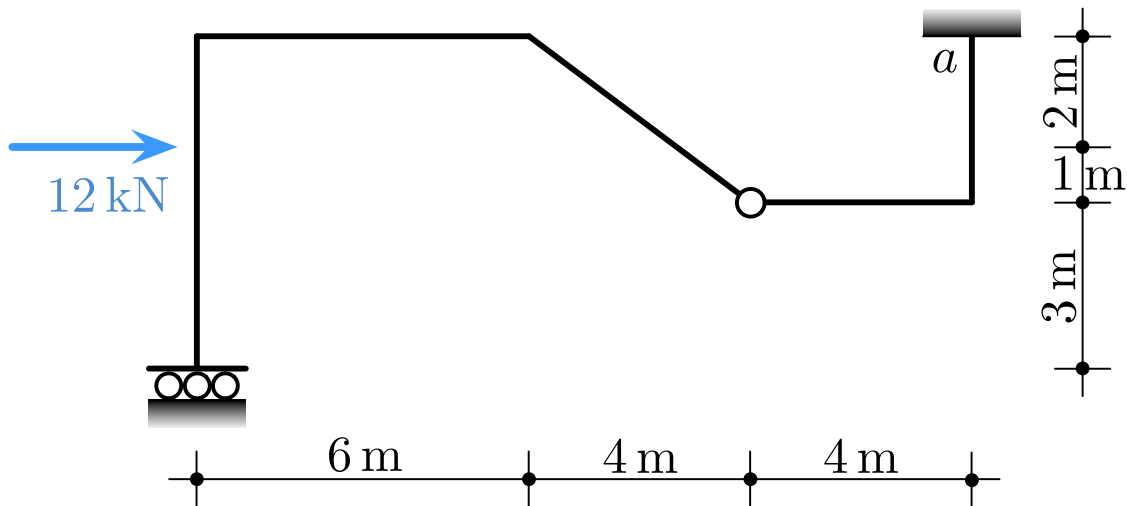
FONDAMENTI DI MECCANICA DELLE STRUTTURE

(docente: G. FORMICA)

PROVA DI VERIFICA – 17 gennaio 2018

STUDENTE:

traccia C



Parte 2

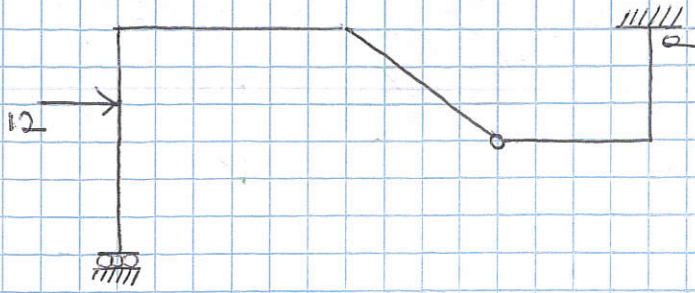
Del sistema iperstatico rappresentato in figura, composto di elementi in acciaio caratterizzati da un momento ultimo $M_u = 500 \text{ kN m}$, si stimi il carico di collasso secondo i teoremi dell'analisi limite. Scelta come incognita X la **reazione a momento dell'incastro** in a , si consegnino

2.1. i risultati ottenuti all'interno dell'approccio statico:

- i diagrammi di (N_0, T_0, M_0) e (N_X, T_X, M_X) distribuiti sullo schema isostatico,
- il valore del fattore di amplificazione del carico λ_s e
- il relativo diagramma $M = M_0 + M_X$ staticamente ammissibile ($|M| \leq M_u$);

2.2. i risultati ottenuti all'interno dell'approccio cinematico:

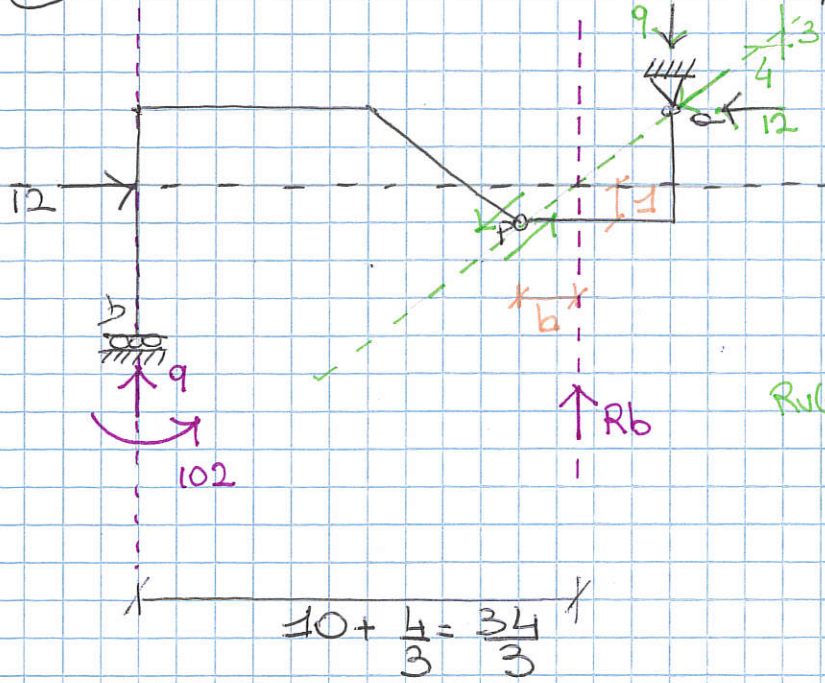
- il (grafico del) meccanismo di collasso cinematicamente ammissibile,
- il relativo valore del fattore di amplificazione del carico λ_p .



SISTEMA STRUTTURALE IPERSTATICO

SISTEMA ISOSTATICO

① SISTEMA ISOSTATICO SCHEMA Ø

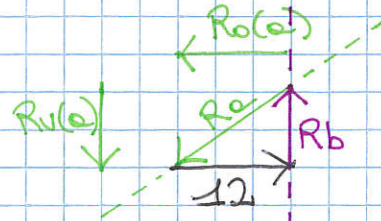


• EQL. DX CASO Ø - 2F

$$R_0(a) = R_0(b)$$

$$R_v(a) = R_v(b)$$

• EQL. SX (GLO CASO 3 - 3Fno//)



$$R_0(a) = 12$$

$$\rightarrow \frac{R_0(a)}{R_v(a)} = \frac{4}{3}$$

$$\rightarrow R_v(a) = 9 = R_b$$

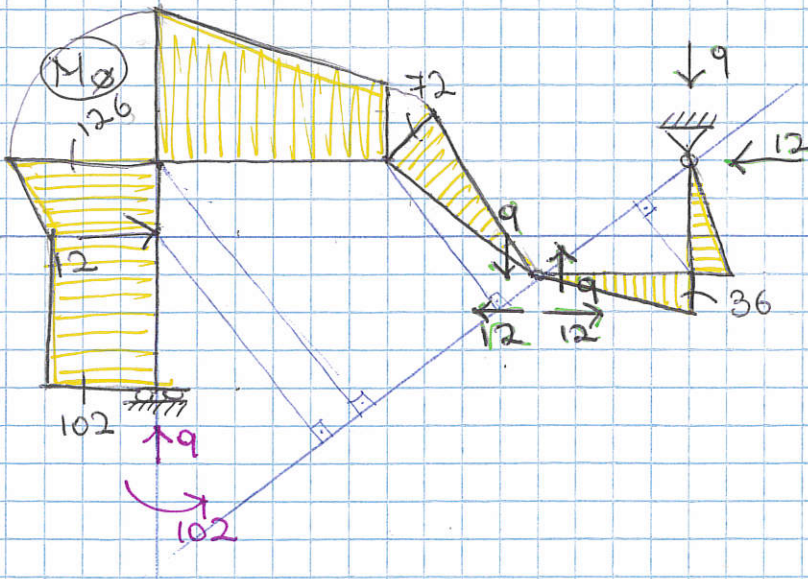
EQV. 2



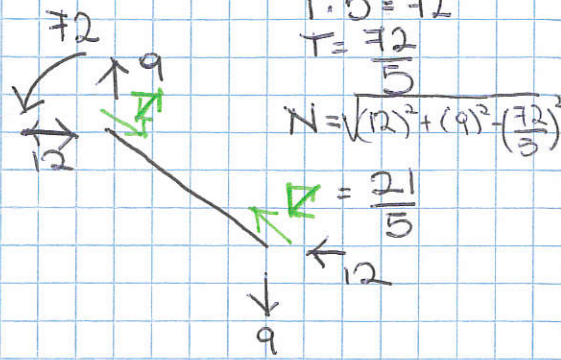
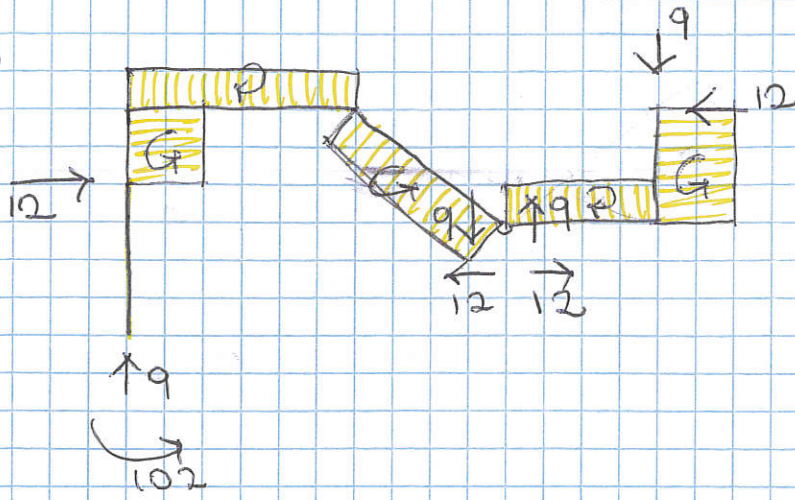
$$\frac{b}{1} = \frac{4}{3}$$

$$R_b = 9$$

$$M_b = 9 \cdot \frac{34}{3} = 102$$



①

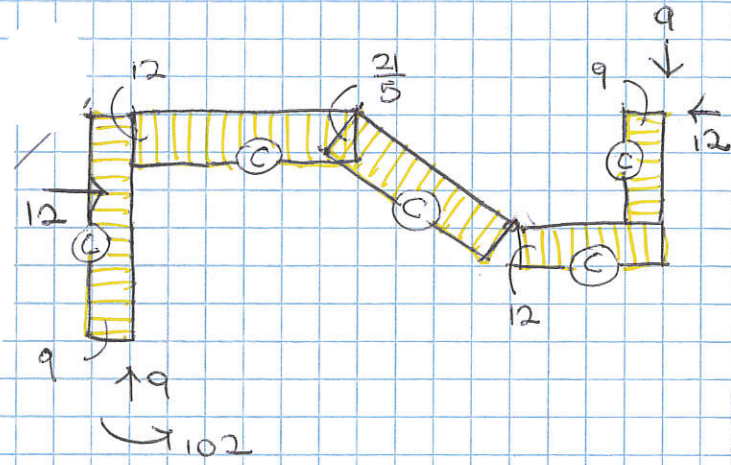


$$T \cdot 5 = 42$$

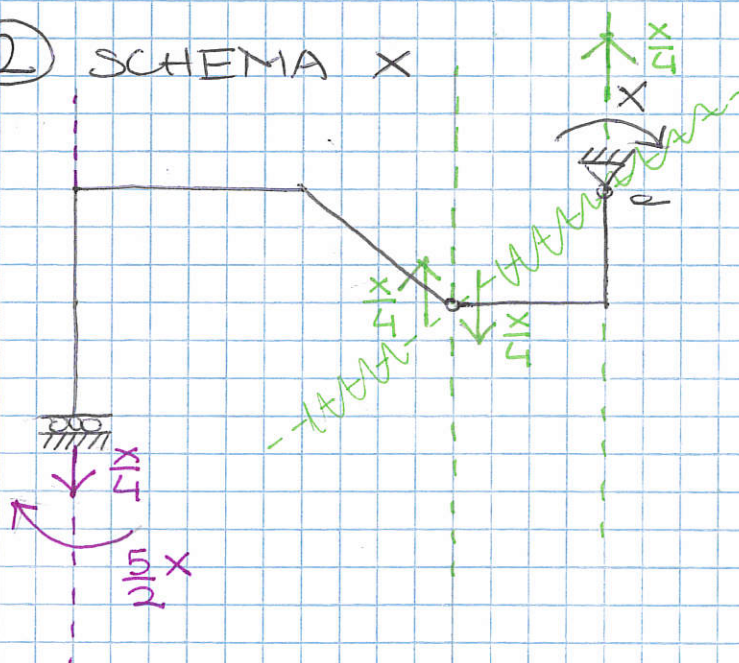
$$T = \frac{42}{5}$$

$$N = \sqrt{(12)^2 + (9)^2} = \frac{72}{5}$$

②



② SCHEMA X



EQL. DX CASO 1 - 2F + 1M

$$R_v \cdot 4 = X$$

$$R_v = \frac{X}{4}$$

EQL. BX CASO 1

$$R_v \cdot 10 = M_b$$

$$\frac{X}{4} \cdot 10 = \frac{5}{2} X = M_b$$

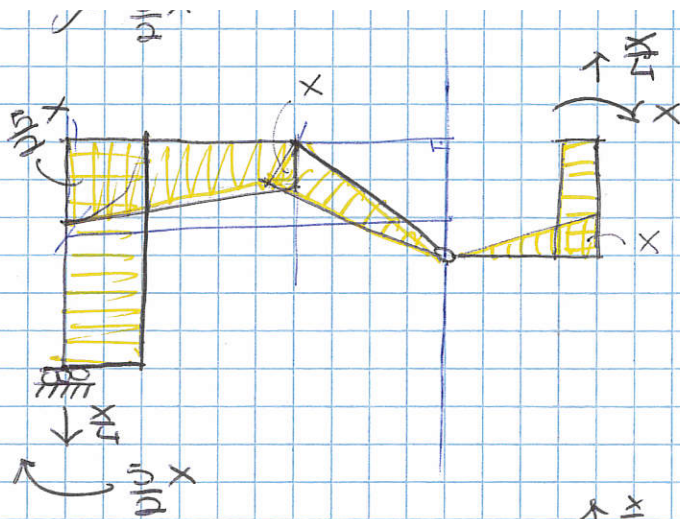
EQL. GLO CASO 1

$$R_v \cdot 14 = M_b + X$$

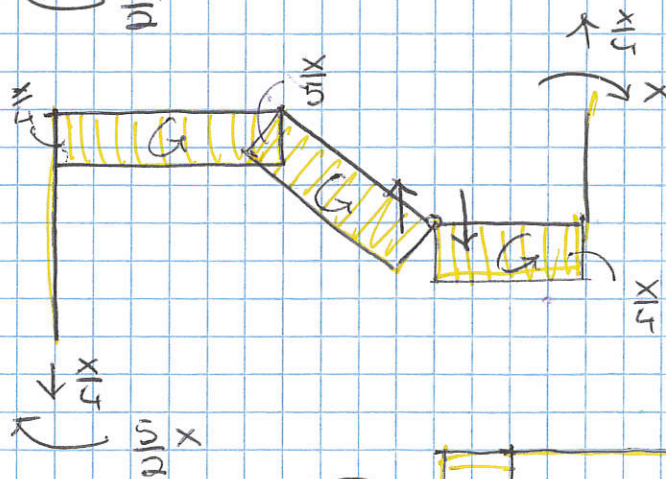
$$\frac{X}{4} \cdot 14 = \frac{5}{2} X + X$$

$$\frac{7}{2} X = \frac{7}{2} X$$

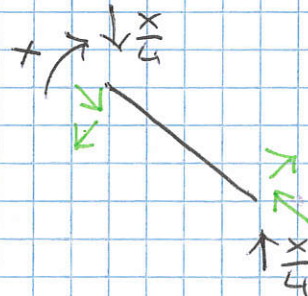
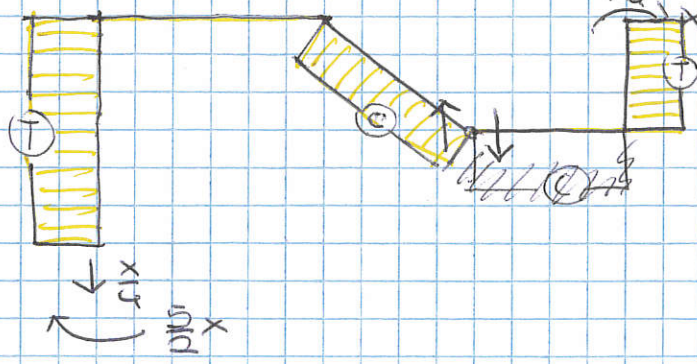
M_x



T_x



N_x



$$T \cdot 5 = X$$

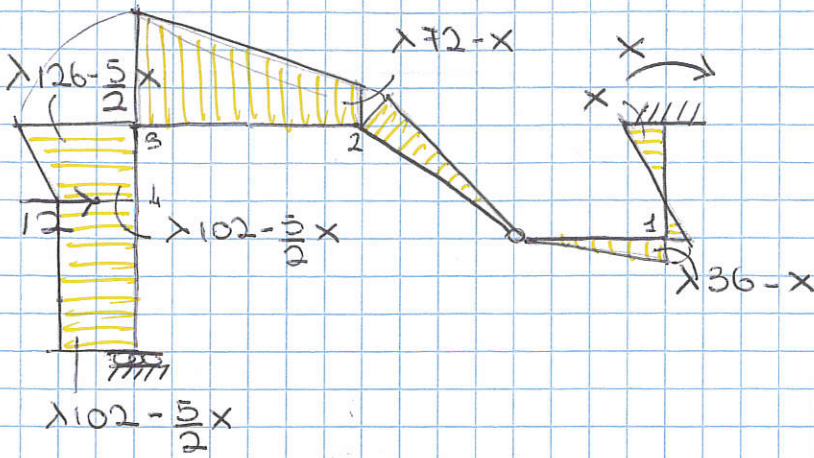
$$T = \frac{X}{5}$$

$$N = \sqrt{\left(\frac{X}{5}\right)^2 + \left(\frac{X}{5}\right)^2}$$

$$= \frac{3}{5} X$$

$$M_u = 500 \text{ kNm}$$

③ $\lambda_D M_0 + M_x$

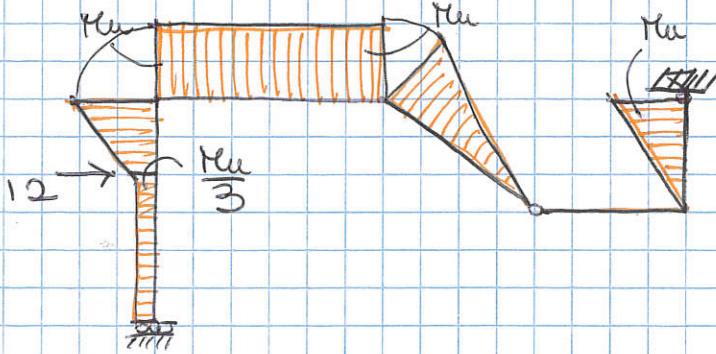


M_0	$x \leq M_u$
M_1	$\lambda 36 - x \leq M_u$
M_2	$\lambda 72 - x \leq M_u$
M_3	$\lambda 126 - \frac{5}{2} x \leq M_u$
M_4	$\lambda 102 - \frac{5}{2} x \leq M_u$

$$\begin{aligned}
 M_1 \rightarrow \lambda_D &= \frac{250}{9} \approx 27,8 \\
 x = M_u \rightarrow M_2 \rightarrow \lambda_D &= \frac{125}{9} \approx 13,9 \\
 M_3 \rightarrow \lambda_D &= \frac{3500}{252} = \frac{125}{9} \approx 13,9 \\
 M_4 \rightarrow \lambda_D &= \frac{3500}{204} = \frac{875}{51} \approx 17,15
 \end{aligned}$$

$$\lambda_D = 13,9$$

④ APPROCCIO STATICO (Mammabile) $\lambda_D = 13,9$



$$x = M_u = M_{M_0} = M_2 = M_3$$

$$M_1 = \frac{125}{9} 36 - 500 = 0$$

$$M_4 = \frac{125}{9} 102 - \frac{5}{2} 500 = \frac{M_u}{3}$$

⑤ APPROCCIO CINEMATICO

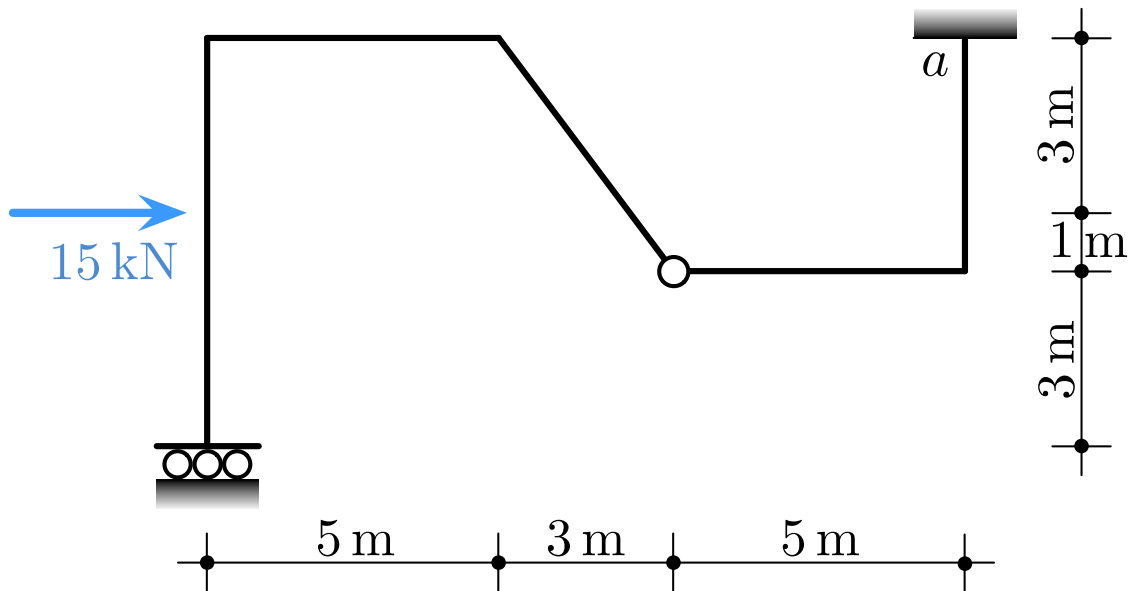
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STUDENTE:

traccia **E**



Parte 2

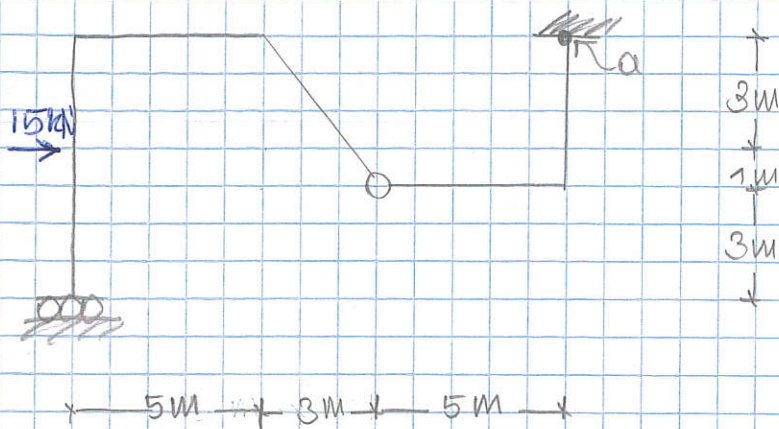
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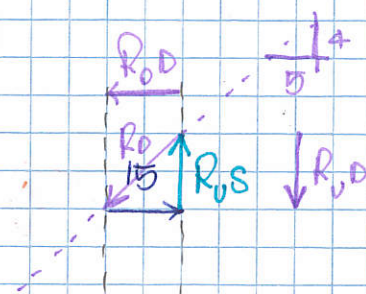
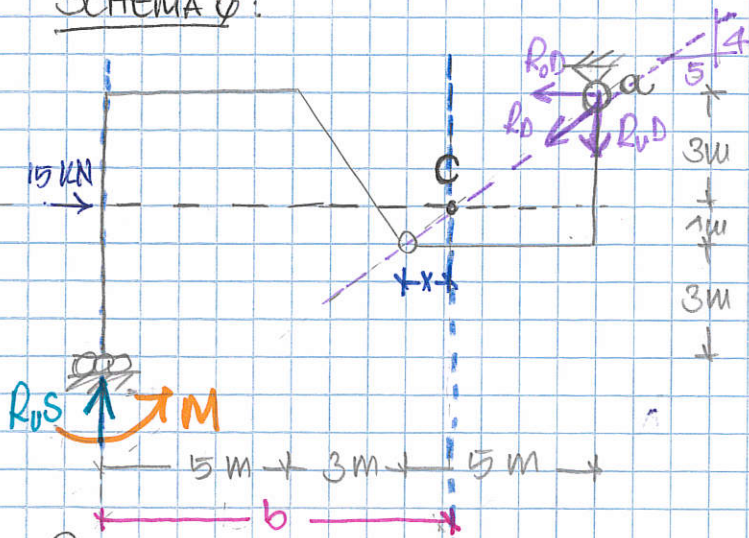
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→ APPROCCIO STATICO:

SCHEMA Ø:



$$R_{0D} = 15; R_{vD} = R_{vS}$$

$$R_{0D} : R_{vD} = 5 : 4$$

$$\Rightarrow R_{0D} = \frac{5}{4} R_{vD}$$

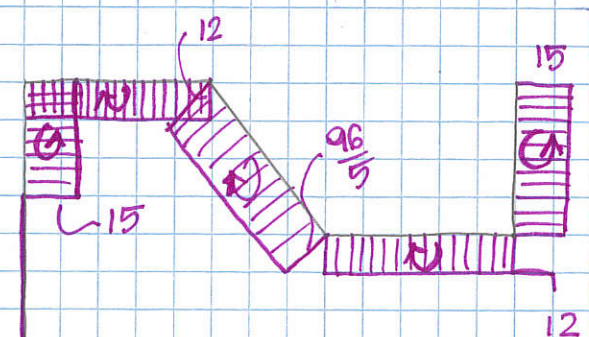
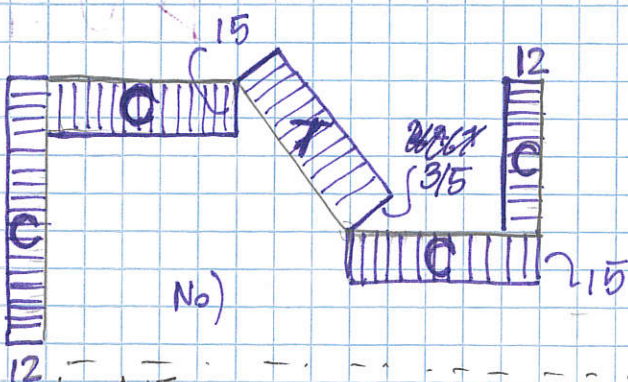
$$R_{0D} = \frac{5}{4} R_{vD} = 15 \Rightarrow \frac{5}{4} R_{vD} = 15 \Rightarrow R_{vD} = \frac{15 \cdot 4}{5} = 12$$

$R_{vD} = 12$ $R_{vS} = 12$ $R_{0D} = 15$

$$b \Rightarrow 5:4 = x:1 \Rightarrow x = \frac{5}{4} \Rightarrow b = 8 + x = 8 + \frac{5}{4} = \frac{32+5}{4} = \frac{37}{4}$$

$$M = 12 \cdot \frac{37}{4} = 111$$

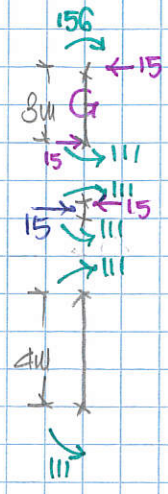
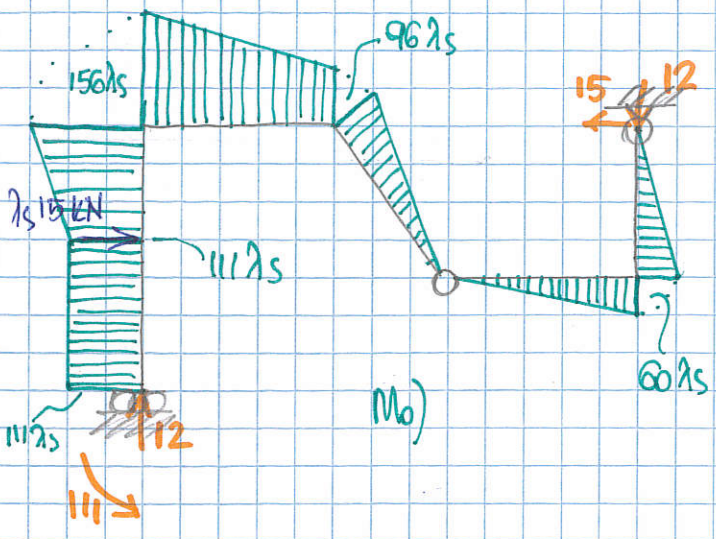
$M = 111$



$$12 \cdot 3 + 15 \cdot 4 = M = 96$$

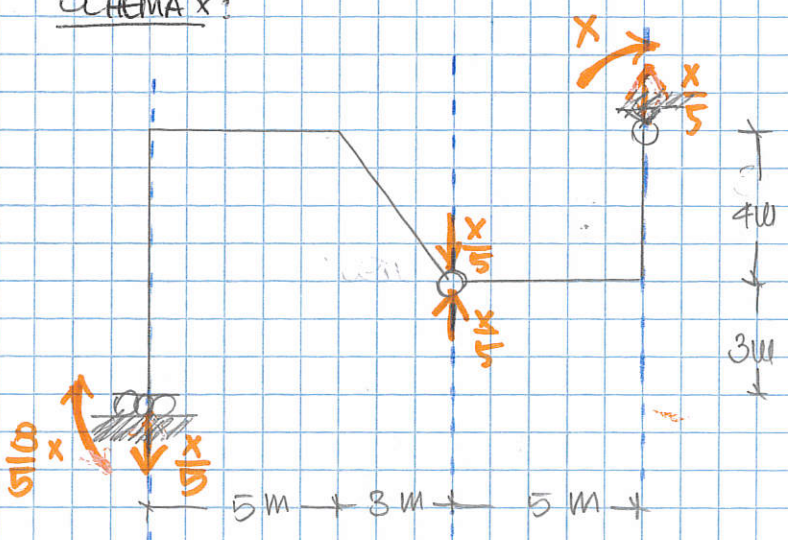
$$T_0 = 96 \Rightarrow T = \frac{96}{5}$$

$$N = \sqrt{12^2 + 15^2 - \left(\frac{96}{5}\right)^2} = \frac{3}{5}$$



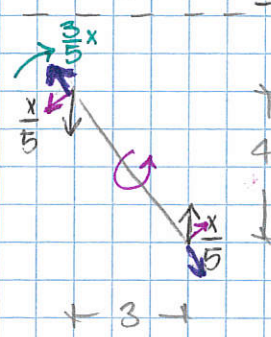
$$M = 15 \cdot 3 + 111 = 45 + 111 = 156$$

SCHEMA X:



EQU. ADX: $x = R \cdot 5 \Rightarrow R = \frac{x}{5}$

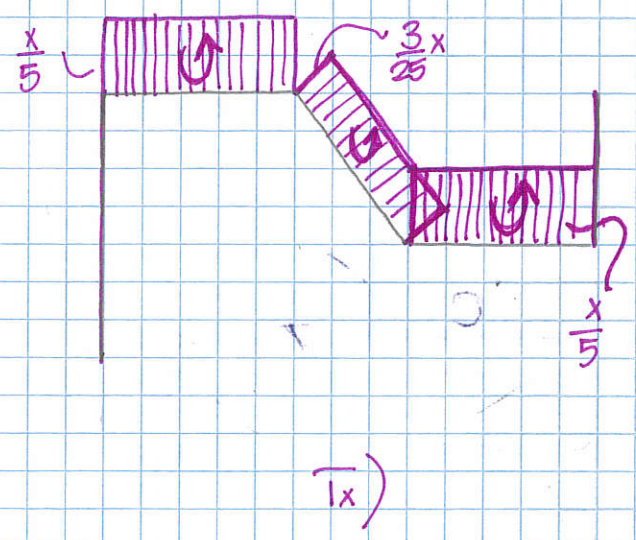
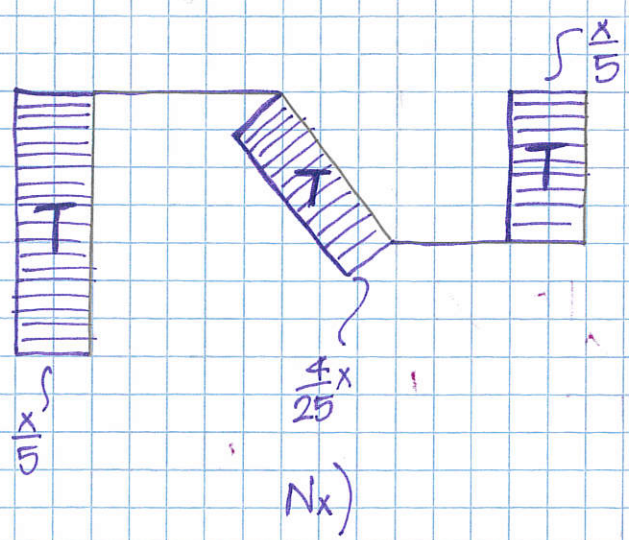
EQU. ASX: $\frac{x}{5} \cdot 8 = M \Rightarrow M = \frac{8x}{5}$

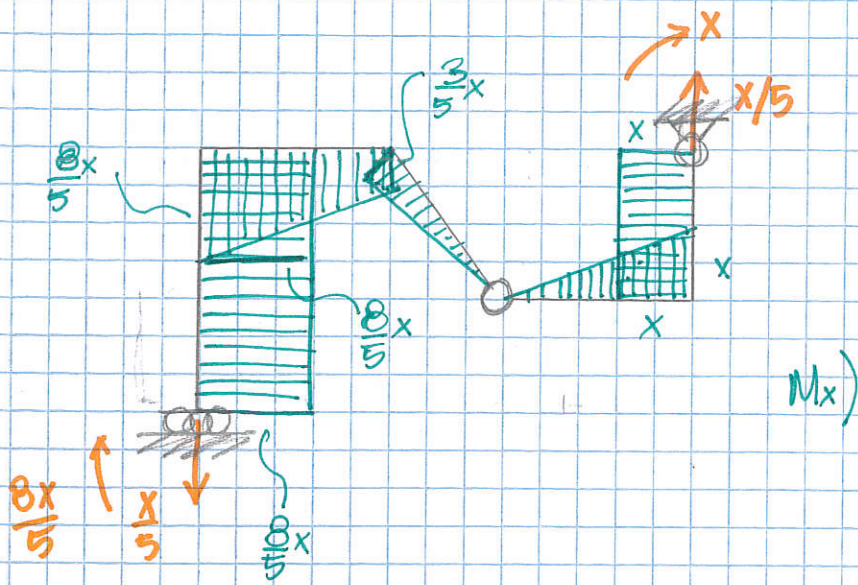


$$\frac{x}{5} \cdot 3 = M \Rightarrow M = \frac{3x}{5}$$

$$T \cdot 5 = \frac{3x}{5} \Rightarrow T = \frac{3x}{25}$$

$$N = x \sqrt{\left(\frac{1}{5}\right)^2 - \left(\frac{3}{25}\right)^2} = \frac{4x}{25}$$



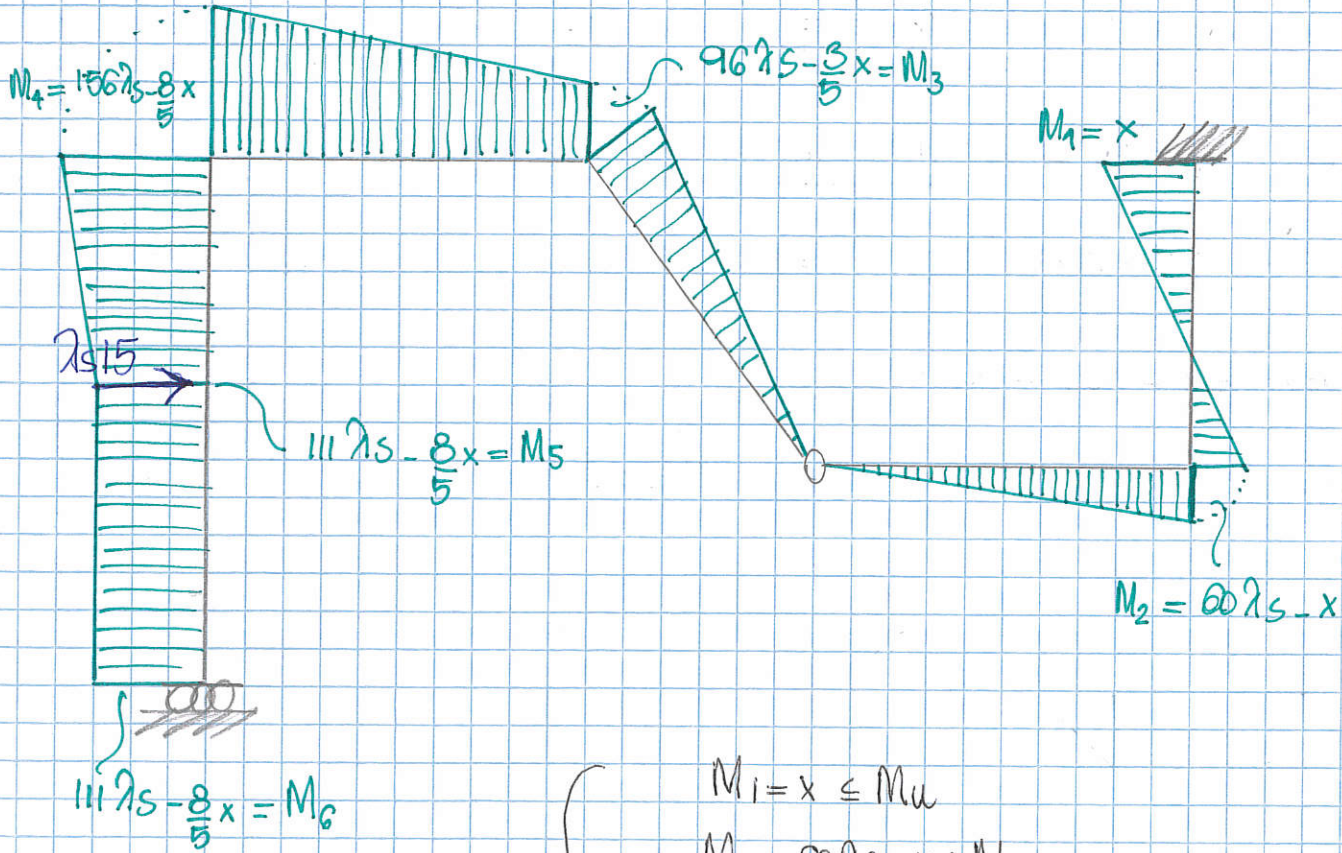


$$M_9 = \frac{8x}{5} \cdot \frac{1}{5} - \frac{x}{5} \cdot \frac{1}{5} = \frac{3x}{5}$$



$M(x)$

$$M = M_0 + M_x$$



$$M_u = 500 \text{ kN}\cdot\text{m}$$

$$\left. \begin{aligned} M_1 &= x \leq M_u \\ M_2 &= 60 \cdot 75/5 - x \leq M_u \\ M_3 &= 96 \cdot 75/5 - \frac{3x}{5} \leq M_u \\ M_4 &= 156 \cdot 75/5 - \frac{8x}{5} \leq M_u \\ M_6 &= M_5 = 111 \cdot 75/5 - \frac{8x}{5} \leq M_u \end{aligned} \right\}$$

1° TENTATIVO : $x = M_u = 500 \text{ kN}\cdot\text{m}$

$$M_2 = 60 \lambda_s - M_u \leq M_u \rightarrow 60 \lambda_s \leq 2M_u \rightarrow \lambda_s \leq \frac{2M_u}{60} \approx 16.66$$

$$M_3 = 96 \lambda_s - \frac{3}{5} M_u \leq M_u \rightarrow 96 \lambda_s \leq \frac{8}{5} M_u \rightarrow \lambda_s \leq \frac{8M_u}{5 \cdot 96} \approx 8.33 \leftarrow$$

$$M_4 = 156 \lambda_s - \frac{8}{5} M_u \leq M_u \rightarrow 156 \lambda_s \leq \frac{13}{5} M_u \rightarrow \lambda_s \leq \frac{13 \cdot M_u}{5 \cdot 156} \approx 8.33 \leftarrow$$

VALORE λ_s
DA PRENDERE
IN CONSIDERA-
ZIONE

$$M_5 = 111 \lambda_s - \frac{8}{5} M_u \leq M_u \rightarrow 111 \lambda_s \leq \frac{13}{5} M_u \rightarrow \lambda_s \leq \frac{13 \cdot M_u}{5 \cdot 111} \approx 11.71$$

Vengono due valori uguali perché semplificando il λ_s relativo a M_3 sarebbe:

$$\lambda_s = \frac{\frac{8}{5} M_u}{5 \cdot 96} = \frac{M_u}{5 \cdot 12}$$

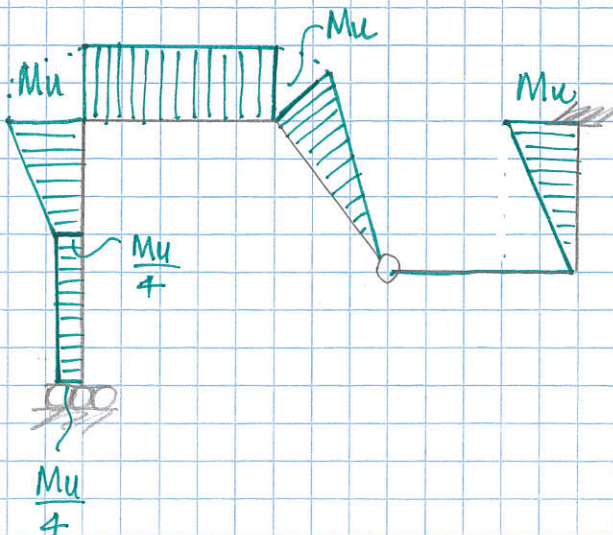
e il λ_s relativo a M_4 è: $\lambda_s = \frac{\frac{13}{5} M_u}{5 \cdot 156} = \frac{M_u}{5 \cdot 12}$

Quindi per la verifica si ha $M_3 = M_4 = M_u = 500 \text{ kN}\cdot\text{m}$.

VERIFICA PER QUATTRO MOMENTI CON $\lambda_s = \frac{M_u}{5 \cdot 12} = \frac{25}{3}$; $x = M_u = 500$

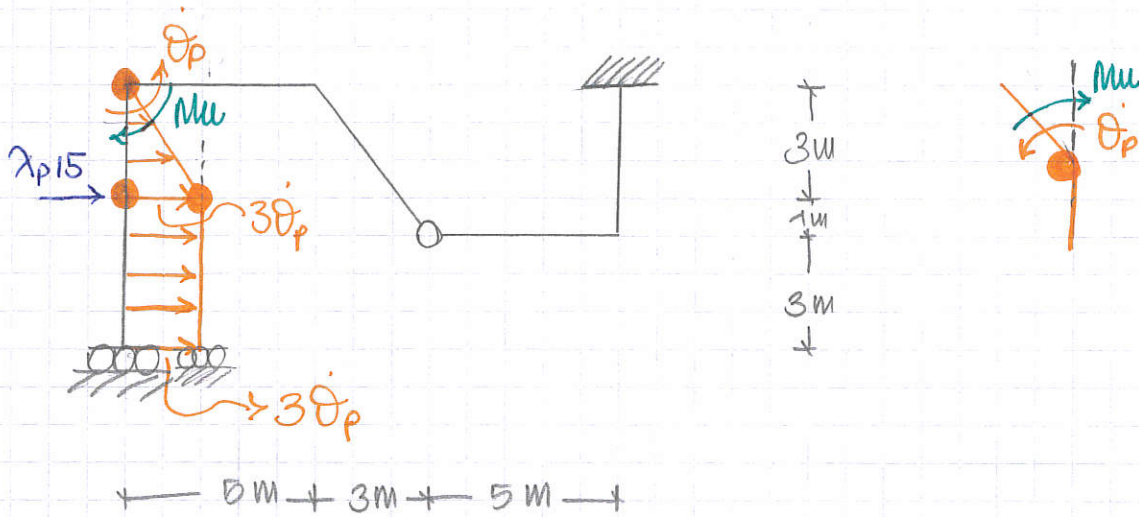
$$M_2 = 60 \cdot \frac{25}{3} - 500 = 500 - 500 = 0$$

$$M_6 = M_5 = 111 \cdot \frac{25}{3} - \frac{8}{5} \cdot 500 = \frac{2295}{3} - 800 = \frac{2295 - (3 \cdot 800)}{3} = \frac{395}{3} = 125 = \frac{M_u}{4}$$



$$\lambda_s = \frac{500}{60} \approx 8.33$$

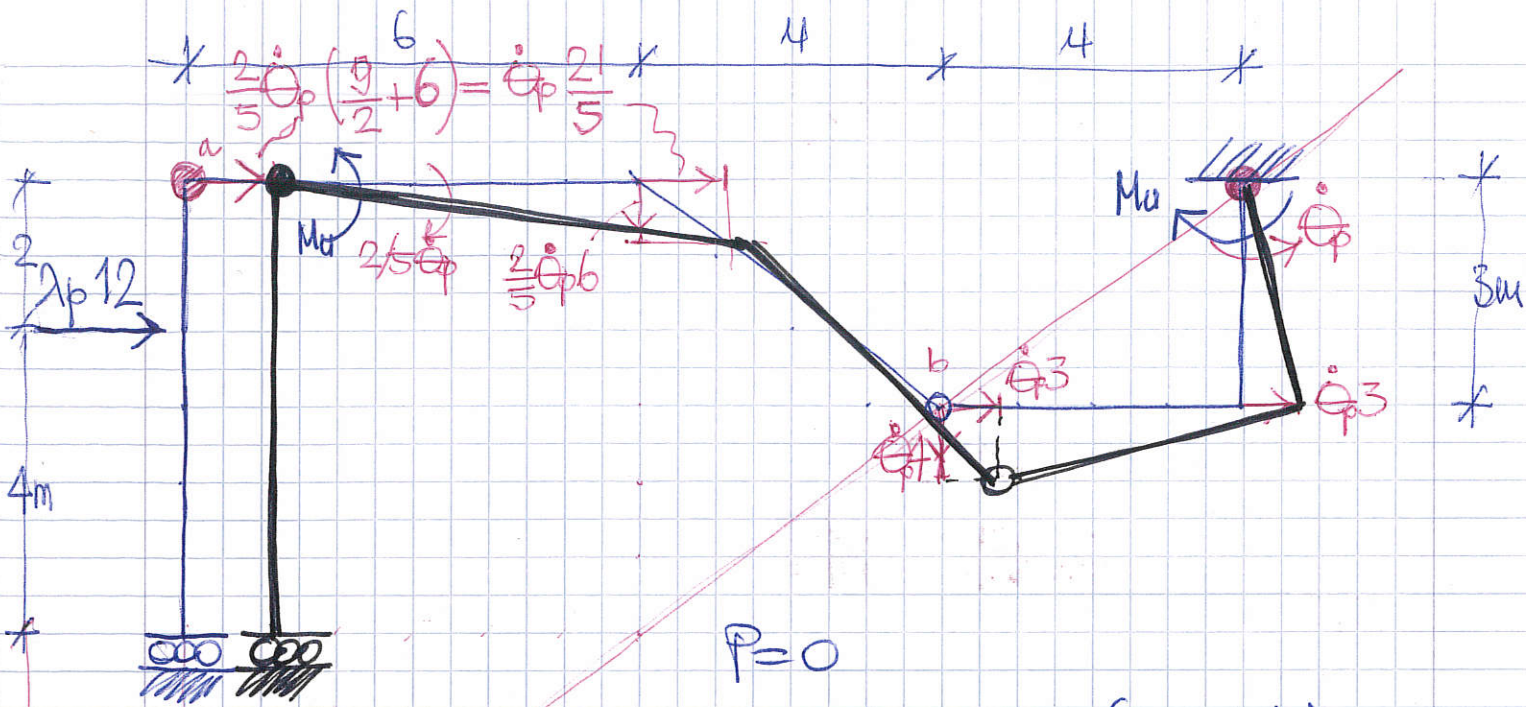
→ APPROCCIO CINEMATICO:



$$Q = 15 \lambda_p \dot{\theta}_p 3 - Mu (\dot{\theta}_p + \dot{\theta}_p) = 0$$

$$15 \lambda_p \dot{\theta}_p 3 - Mu 2 \dot{\theta}_p = 0 \Rightarrow \lambda_p = \frac{Mu 2}{15 \cdot 3} = \frac{500 \cdot 2}{15 \cdot 3} \approx 22.22$$

$$\lambda_s \leq \lambda_c \leq \lambda_p \Rightarrow 8.33 \leq \lambda_c \leq 22.22$$

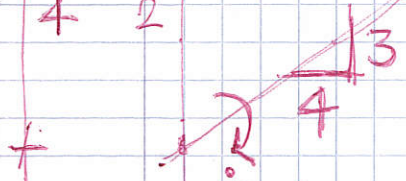


$P=0$

$$\lambda_p \cdot 12 \dot{\theta}_p \frac{21}{5} - M_u \left(\dot{\theta}_p + \frac{2}{5} \dot{\theta}_p \right) = 0$$

$$\rightarrow \lambda_p = \frac{M_u \cdot 7/5}{12 \times \frac{21}{5}} = \frac{500^{125}}{369} = \frac{125}{9} \approx 13.9$$

$$\frac{6 \cdot 3}{4} = \frac{9}{2}$$



$$\frac{\dot{\theta}_p \cdot 4}{10} = \frac{2}{5} \dot{\theta}_p \quad (\text{rotazione corpo tra a-b})$$

$$\lambda_s = \lambda_p = \boxed{13.9 = \lambda_c}$$